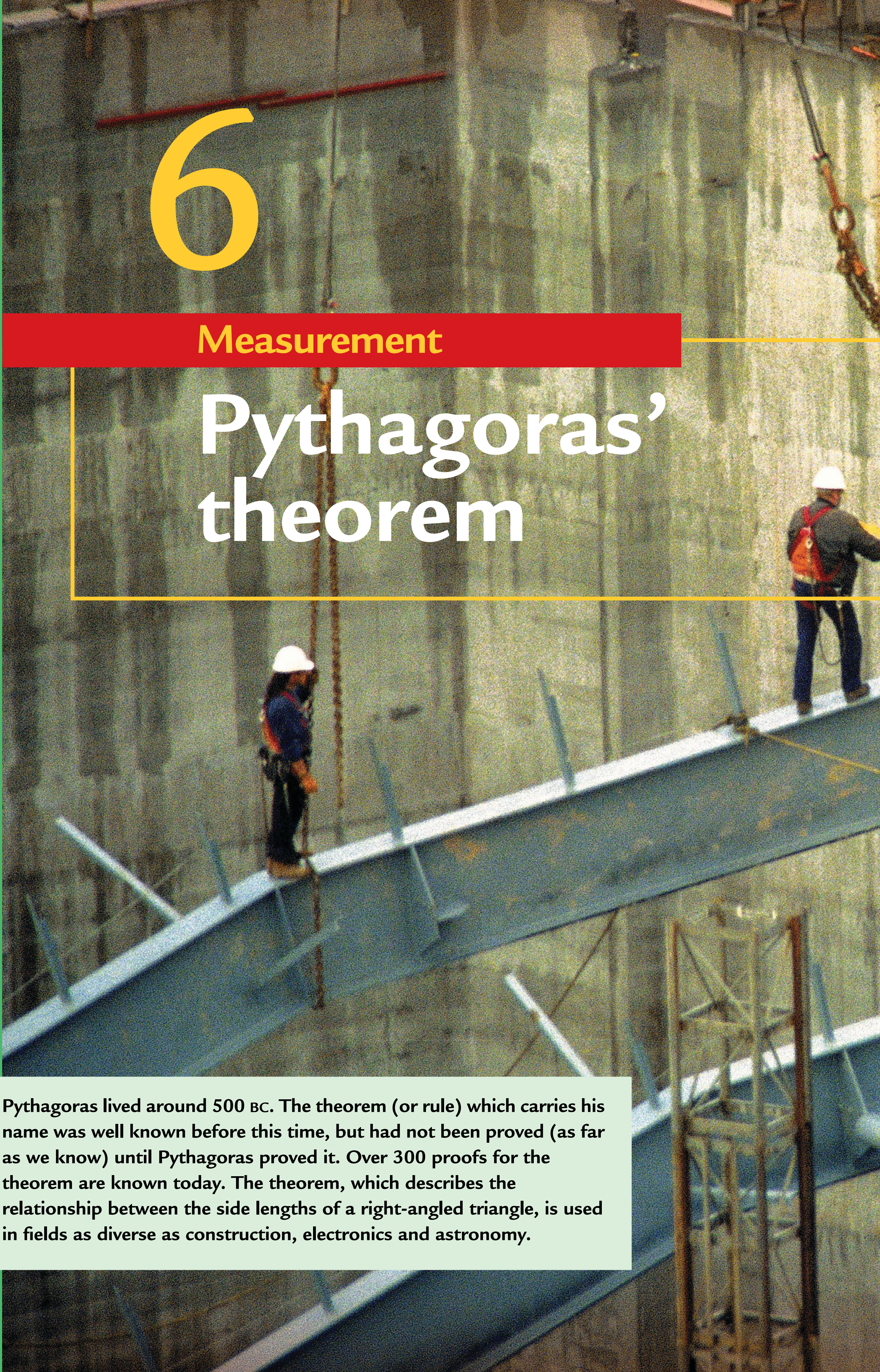


6

Measurement

Pythagoras' theorem

Pythagoras lived around 500 BC. The theorem (or rule) which carries his name was well known before this time, but had not been proved (as far as we know) until Pythagoras proved it. Over 300 proofs for the theorem are known today. The theorem, which describes the relationship between the side lengths of a right-angled triangle, is used in fields as diverse as construction, electronics and astronomy.

In this chapter you will:

- calculate squares and square roots of numbers
- identify the hypotenuse as the longest side of any right-angled triangle
- establish the relationship between the length of the sides of a right-angled triangle in a practical way
- use Pythagoras' theorem to find the length of the hypotenuse or one of the shorter sides in a right-angled triangle
- use Pythagoras' theorem to establish whether a triangle has a right angle
- use Pythagoras' theorem to solve practical problems involving right-angled triangles
- graph two points to form an interval on the number plane and form a right-angled triangle with that interval as the hypotenuse
- use Pythagoras' theorem to calculate the length of an interval on the number plane.

Wordbank

- **hypotenuse** The longest side of a right-angled triangle. It is the side opposite the right angle.
- **theorem** A mathematical rule or law that can be proved.
- **diagonal** An interval from one vertex (corner) of a shape to another non-adjacent vertex.
- **interval** A section of a line, having a definite length.
- **exact length** A length that is written as an exact value, and is not rounded to decimal places.
- **Cartesian plane** Another name for the number plane.

Think!

What is the maximum length of a steel rod that will fit inside a rectangular box whose base dimensions are 70 cm × 50 cm and whose height is 190 cm? How could the maximum length of the rod be determined?

Worksheet 6-01

Brainstarters 6

Start up

1 Convert each of these lengths to metres:

a 500 cm **b** 2000 mm **c** 4 km
d 6000 mm **e** 150 cm **f** 1200 cm
g 6.7 km **h** 1800 mm **i** 85 cm

2 Evaluate:

a 7^2 **b** 12^2 **c** $9^2 + 4^2$
d $\sqrt{121}$ **e** $8^2 + 6^2$ **f** $\sqrt{576}$
g $12^2 - 4^2$ **h** $\sqrt{100 + 576}$ **i** $\sqrt{37^2 - 35^2}$
j $(\sqrt{2})^2$ **k** $(\sqrt{7})^2$ **l** $\sqrt{50^2 - 40^2}$

3 Calculate, correct to one decimal place:

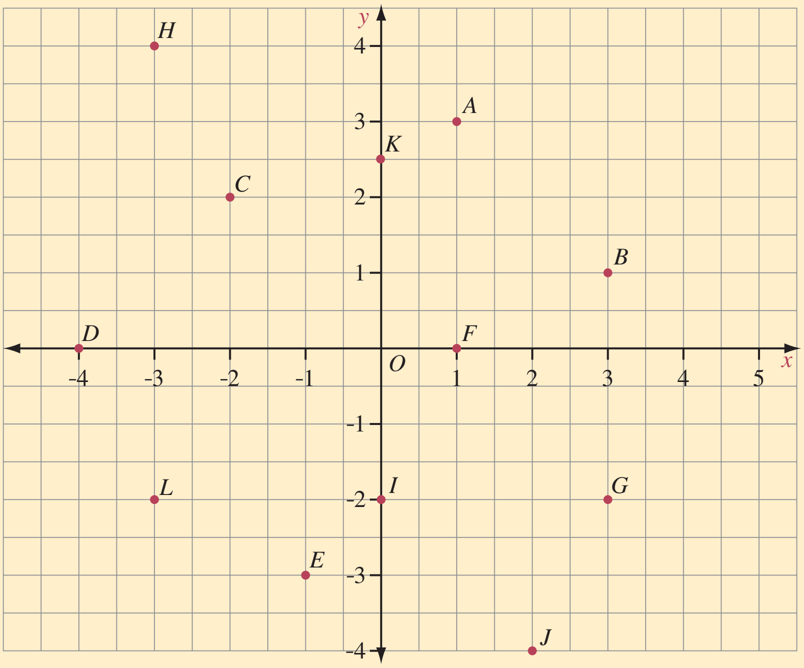
a $\sqrt{75}$ **b** $\sqrt{160}$ **c** $\sqrt{94}$
d $\sqrt{200}$ **e** $\sqrt{787}$ **f** $\sqrt{22\,500}$

4 Solve these equations by finding the value of the pronumeral each time:

a $40 = c + 27$ **b** $18 = y + 6$ **c** $24 = m - 16$ **d** $145 = y - 20$

5 **a** If $p^2 = 36$, find p if p is positive. **b** If k is positive, and $k^2 = 100$, find k.

6 Write the coordinates of the points A, B, C … L on this number plane:



Squares and square roots

The **square** of a number is that number multiplied by itself. For example, the square of 5, written 5^2, is $5 \times 5 = 25$.

The **square root** of a number is the number which, when multiplied by itself, results in the given number. For example, the square root of 9 (written as $\sqrt{9}$) is 3, because $3 \times 3 = 9$.

Example 1

Find the value of:

a 17^2 **b** 9.7^2 **c** $4^2 + 7^2$

Solution

a $17^2 = 289$

17 [x^2] [=] (on a calculator)

b $9.7^2 = 94.09$

9.7 [x^2] [=] (on a calculator)

c $4^2 + 7^2 = 16 + 49 = 65$

4 [x^2] [+] 7 [x^2] [=] (on a calculator)

Example 2

Find the value (correct to three significant figures, where necessary) of:

a $\sqrt{1024}$ **b** $\sqrt{83}$ **c** $\sqrt{5^2 + 7^2}$

Solution

a $\sqrt{1024} = 32$

[$\sqrt{}$] 1024 [=] (on a calculator)

b $\sqrt{83} = 9.110\,433 \approx 9.11$

[$\sqrt{}$] 83 [=] (on a calculator)

c $\sqrt{5^2 + 7^2} = 8.602\,325\ldots \approx 8.60$

5 [x^2] [+] 7 [x^2] [=] [$\sqrt{}$] [=]

or [$\sqrt{}$] [(] 5 [x^2] [+] 7 [x^2] [)] [=] (on a calculator)

Exercise 6-01

1 Find the value of: Example 1

a 7^2	**b** 3^2	**c** 15^2	**d** $(6.2)^2$
e $(31.7)^2$	**f** $(\frac{1}{2})^2$	**g** $(\frac{2}{5})^2$	**h** $(\frac{5}{4})^2$
i $(0.1)^2$	**j** $(0.71)^2$	**k** $3^2 + 5^2$	**l** $20^2 - 7^2$
m $9^2 + 40^2$	**n** $5^2 + 18^2$	**o** $50^2 - 40^2$	**p** $(3.2)^2 - (1.5)^2$
q $(1.2)^2 + (0.7)^2$	**r** $(0.3)^2 + (0.4)^2$	**s** $(2\frac{1}{2})^2 + 3^2$	**t** $(\frac{1}{4})^2 + (\frac{1}{5})^2$

2 Find the value of each of the following (correct to two decimal places where necessary): Example 2

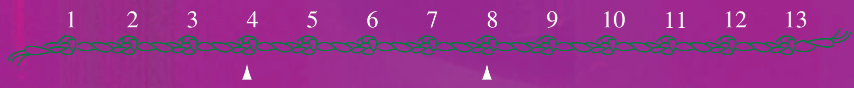
a $\sqrt{144}$	**b** $\sqrt{25}$	**c** $\sqrt{\frac{81}{16}}$	**d** $\sqrt{1.21}$
e $\sqrt{15}$	**f** $\sqrt{900}$	**g** $\sqrt{170}$	**h** $\sqrt{42.7}$
i $\sqrt{0.04}$	**j** $\sqrt{24}$	**k** $\sqrt{7.8}$	**l** $\sqrt{3^2 + 4^2}$
m $\sqrt{20^2 - 15^2}$	**n** $\sqrt{(\frac{1}{2})^2 + (\frac{3}{4})^2}$	**o** $\sqrt{(0.6)^2 + (0.1)^2}$	**p** $\sqrt{(4.5)^2 - 3^2}$
q $\sqrt{(\frac{5}{2})^2 - (\frac{3}{2})^2}$	**r** $\sqrt{(8.2)^2 - (3.5)^2}$	**s** $\sqrt{(4.1)^2 - (0.9)^2}$	**t** $\sqrt{(2\frac{1}{2})^2 + 1^2}$

Just for the record

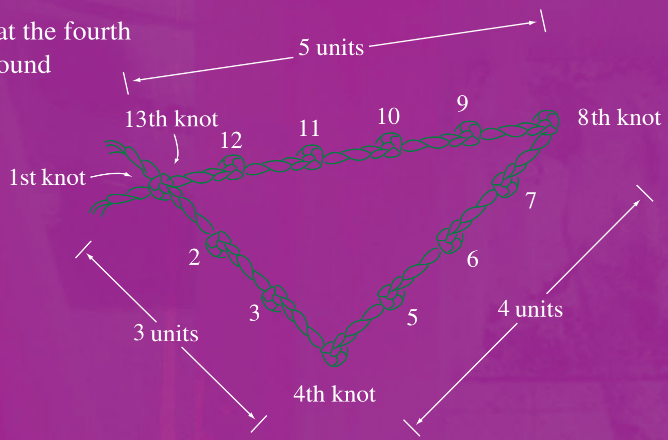
Rope stretchers

In ancient Egypt most of the farms were built along the Nile River. Whenever the Nile flooded, the farmers were faced with rebuilding their fences. They could make the straight sections of fencing quite simply, but needed a method to construct the right angles at the corners of the fields. They used a technique established by a group of Egyptians called *rope stretchers*.

The rope stretchers took a length of rope with thirteen knots tied in it at equal intervals, as shown.



The rope was fixed to the ground at the fourth and eighth knots, and stretched around so that the first knot aligned with the thirteenth knot, as shown. The right angle is formed at the fourth knot.



The Egyptian farmers could not understand why this procedure produced a right angle. The reason was eventually discovered by the Greek mathematician Pythagoras (580–496 BC).

Triangles

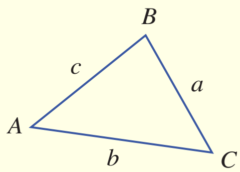
Worksheet 6-02 Pythagoras' discovery

Worksheet 6-03 A page of right-angled triangles

Naming the sides in a triangle

For triangle *ABC* the lengths of the sides of the triangle are usually named in the following manner:

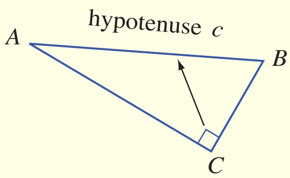
- the side opposite $\angle A$ is called a
- the side opposite $\angle B$ is called b
- the side opposite $\angle C$ is called c



The hypotenuse

In any right-angled triangle, the longest side is called the **hypotenuse**.

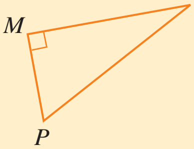
The hypotenuse in the diagram is AB (or BA) , or c because it is opposite the right angle $\angle C$.



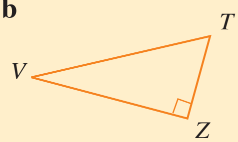
Exercise 6-02

1 Name the hypotenuse in each of the following right-angled triangles.

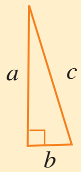
a



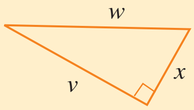
b



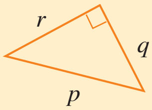
c



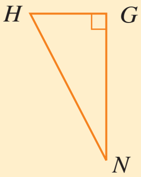
d



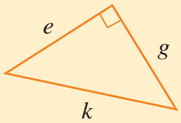
e



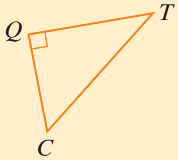
f



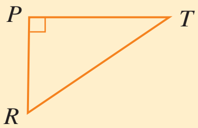
g



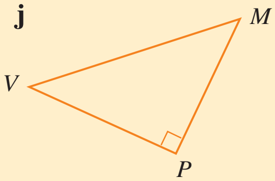
h



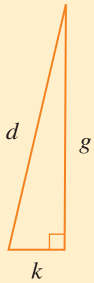
i



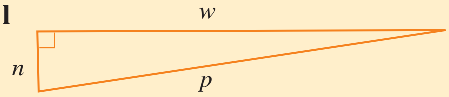
j



k

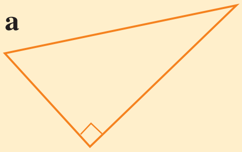


l

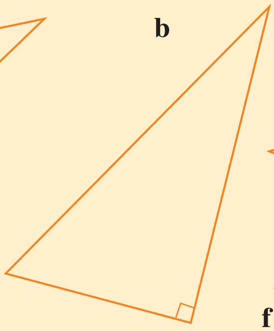


2 Measure the hypotenuse in each of these triangles. Give your answer in millimetres (mm).

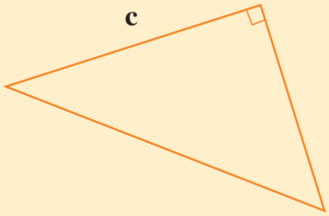
a



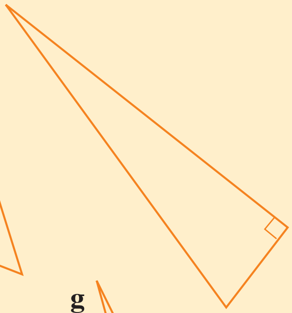
b



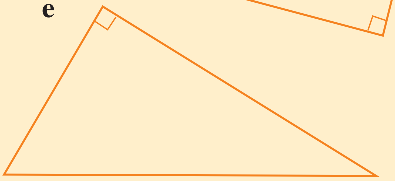
c



d



e



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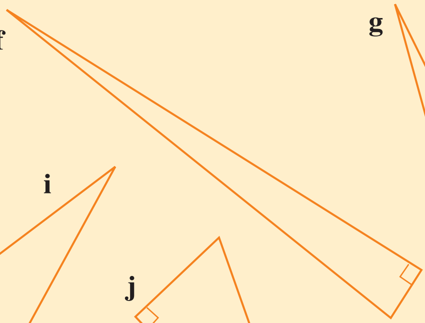

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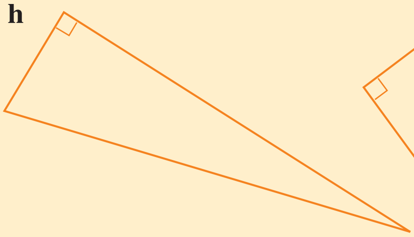

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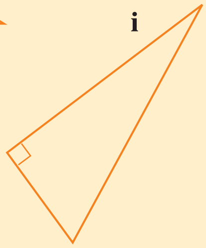

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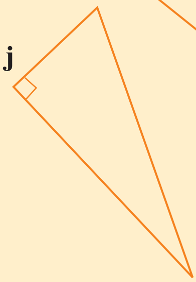


Working mathematically

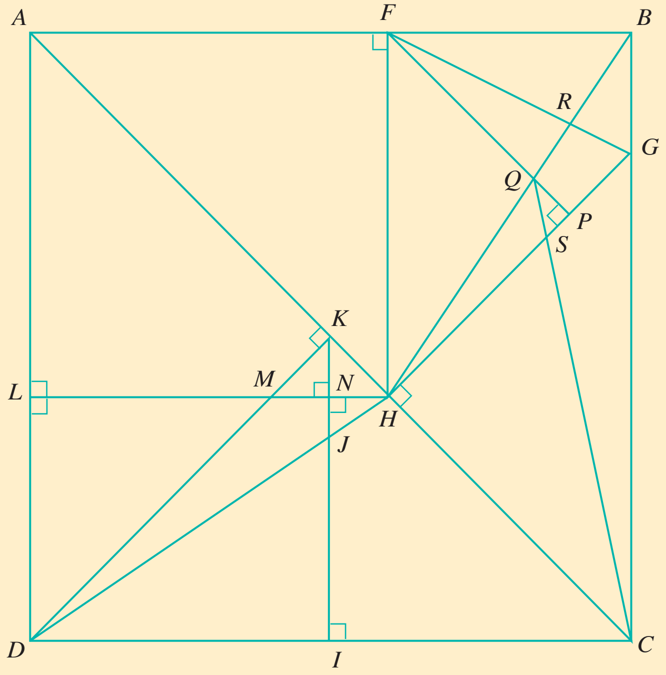
Communicating: Hidden hypotenuses

Work in pairs to complete this investigation.

The square *ABCD*, shown below, has been divided into many shapes, some of which are right-angled triangles.

a List as many hypotenuses as you can find in 10 minutes, writing your answers in the form '*AC* in ΔABC'.

b Share your list with the whole class to find all the hidden hypotenuses.

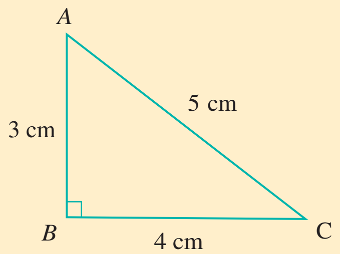


Working mathematically

Applying strategies and reasoning: A rule relating the sides of a right-angled triangle

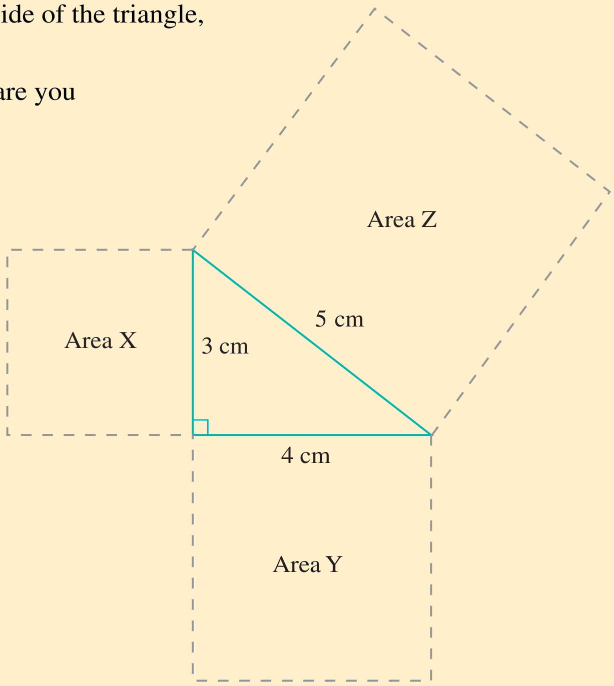
You will need: a protractor, a calculator, a ruler and a pencil.

Step 1: In the centre of a new page accurately construct the right-angled triangle *ABC* shown below.
(Diagrams not to scale.)



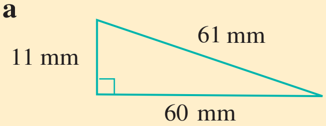
Step 2: Construct a square on each side of the triangle, as shown.

1 **a** Calculate the area of each square you constructed.
b Add together the areas of the squares on the two shorter sides.
c Compare your result from part **b** to the area of the square on the hypotenuse.
d Describe the pattern
$3^2 + 4^2 = \ldots$

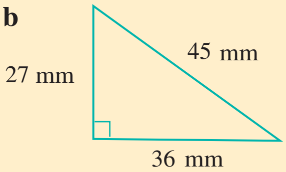


2 Use your calculator to verify that the result you described in Question **1** part **d** is true for these right-angled triangles:

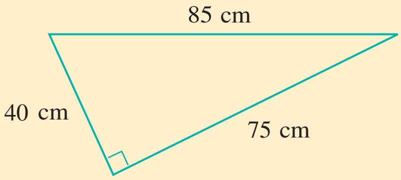
a



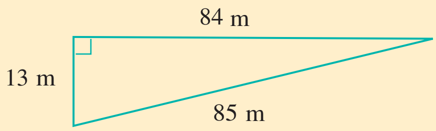
b



c



d



What is Pythagoras' theorem?

What Pythagoras discovered about the sides of a right-angled triangle is a rule called **Pythagoras' theorem**. A theorem is a mathematical rule or law that can be proved.

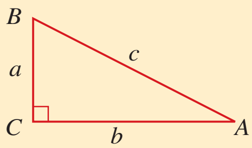
Pythagoras' theorem is as shown in the box below.

Worksheet 6-02
Pythagoras' discovery

In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

As a formula, this is written:

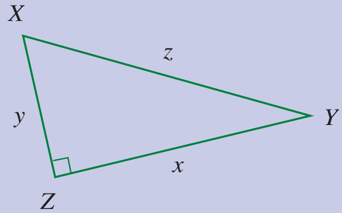
$c^2 = a^2 + b^2$



Worksheet 6-03
A page of right-angled triangles

Example 3

State Pythagoras' theorem for triangle XYZ, shown on the right.



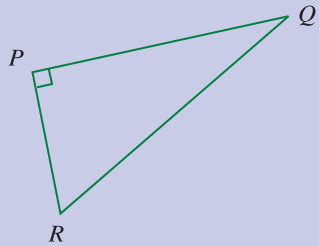
Solution

Pythagoras theorem for ΔXYZ is $z^2 = x^2 + y^2$

Example 4

Choose the correct statement of Pythagoras' theorem for triangle PQR shown at the right:

A $PR^2 = PQ^2 + QR^2$
B $QR^2 = RP^2 + PQ^2$
C $PQ^2 = RP^2 + QR^2$



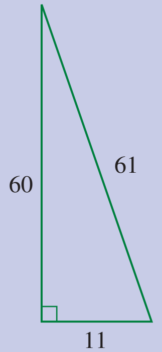
Solution

The correct statement is statement **B**, (that is, $QR^2 = RP^2 + PQ^2$).

Example 5

Choose the correct statement of Pythagoras' theorem for ΔABC shown.

A $61^2 = 11^2 + 60^2$
B $60^2 = 61^2 + 11^2$
C $11^2 = 60^2 + 61^2$
D $60 = 11 + 61$



Solution

The correct statement is statement **A** (that is, $61^2 = 11^2 + 60^2$).

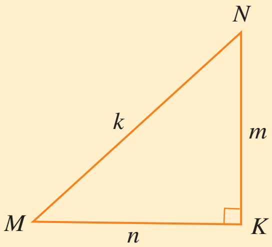
Exercise 6-03

Example 3

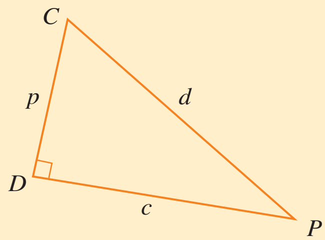
SkillBuilder 21-02 Pythagoras' theorem

1 State Pythagoras' theorem as it applies to each of the following right-angled triangles.

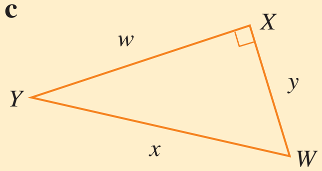
a



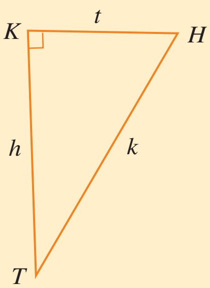
b



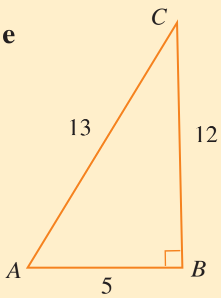
c



d



e



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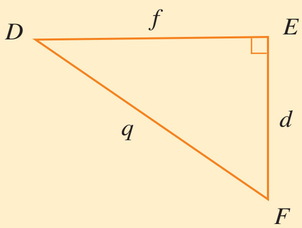
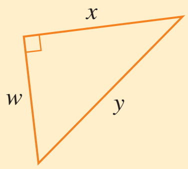


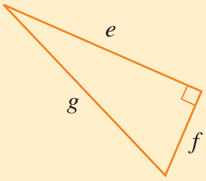
2 Choose the correct statement of Pythagoras' theorem for each triangle shown below.

Example 4

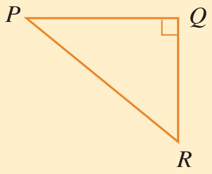
a
A $y^2 = x^2 + w^2$
B $x^2 = w^2 + y^2$
C $w^2 = x^2 + y^2$
D $y = x + w$



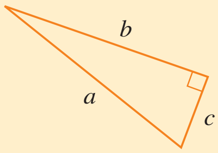
b
A $e^2 = f^2 + g^2$
B $g = e + f$
C $f^2 = e^2 + g^2$
D $g^2 = e^2 + f^2$



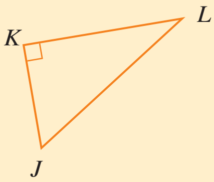
c
A $PR = PQ + QR$
B $QR^2 = PQ^2 + PR^2$
C $RP^2 = RQ^2 + PQ^2$
D $PQ^2 = PR^2 + QR^2$



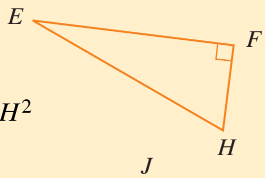
d
A $a^2 + c^2 = b^2$
B $c^2 = a^2 + b^2$
C $a^2 = b^2 + c^2$
D $b^2 = a^2 + c^2$



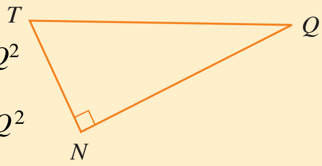
e
A $KL^2 + KJ^2 = LJ^2$
B $JL^2 + KL^2 = KJ^2$
C $KL^2 = JK^2 + JL^2$
D $K + J + L = 0$



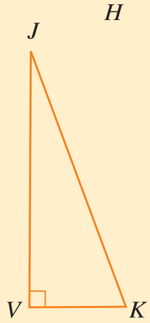
f
A $f^2 = e^2 + h^2$
B $h^2 = f^2 + e^2$
C $e^2 = f^2 + h^2$
D $FH^2 = EF^2 + EH^2$



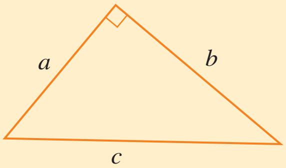
g
A $q^2 = n^2 + t^2$
B $TN^2 + TQ^2 = NQ^2$
C $n^2 = q^2 + t^2$
D $TN^2 = TQ^2 + NQ^2$



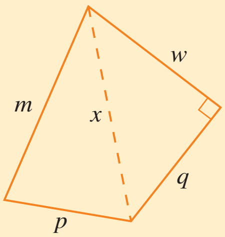
h
A $VK^2 + VJ^2 = JK^2$
B $j^2 = v^2 + k^2$
C $v = j^2 + k^2$
D $VK^2 = JV^2 + JK^2$



i
A $a^2 = b^2 + c^2$
B $b^2 = a^2 + c^2$
C $c^2 = a^2 + b^2$
D none of these



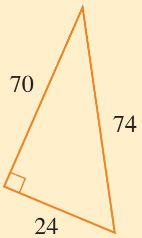
j
A $x^2 = m^2 + p^2$
B $w^2 = q^2 + x^2$
C $m^2 = q^2 + w^2$
D $x^2 = q^2 + w^2$



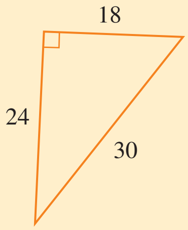
3 Choose the correct statement of Pythagoras' theorem for each triangle shown below:

Example 5

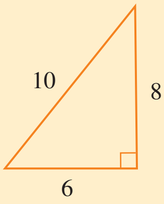
a
A $74^2 + 70^2 = 20^2$
B $24^2 + 70^2 = 74^2$
C $70 = 20 + 74$
D $20^2 + 74^2 = 70^2$



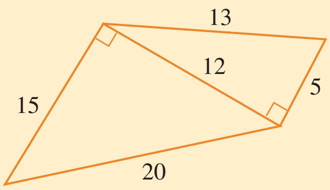
b
A $30 = 18 + 24$
B $18^2 + 30^2 = 24^2$
C $18^2 = 24^2 + 30^2$
D $30^2 = 18^2 + 24^2$



c
A $10^2 + 8^2 = 6^2$
B $10 + 8 = 6$
C $6^2 + 8^2 = 10^2$
D $6 + 8 = 10$



d
A $15^2 + 13^2 = 20^2$
B $5^2 + 12^2 = 13^2$
C $5^2 + 12^2 = 20^2$
D $12^2 + 15^2 = 20^2$



Worksheet 6-04
Proving Pythagoras' theorem

Geometry 6-01
Measuring Pythagoras

Geometry 6-02
Pythagoras dissection

Geometry 6-03
Perigal's proof

Geometry 6-04
Leonardo's proof

Working mathematically

Applying strategies and reasoning: Proving Pythagoras' theorem

You will need: tracing (or blank) paper, scissors, glue.

How can you prove that, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides?

Proof 1

One way is to cut out the squares on the two shorter sides and assemble them to completely cover the square on the hypotenuse.

Step 1: Make two copies of Figure A.

Step 2: Cut out the first copy of Figure A and paste it into your workbook, keeping all the squares intact.

Step 3: Cut out the second copy of Figure A. Colour the squares if you wish. Then cut the squares on the two shorter sides into the five numbered pieces shown.

Step 4: Arrange the five pieces to exactly cover the square on the hypotenuse. Paste this proof of Pythagoras' theorem into your workbook as well.

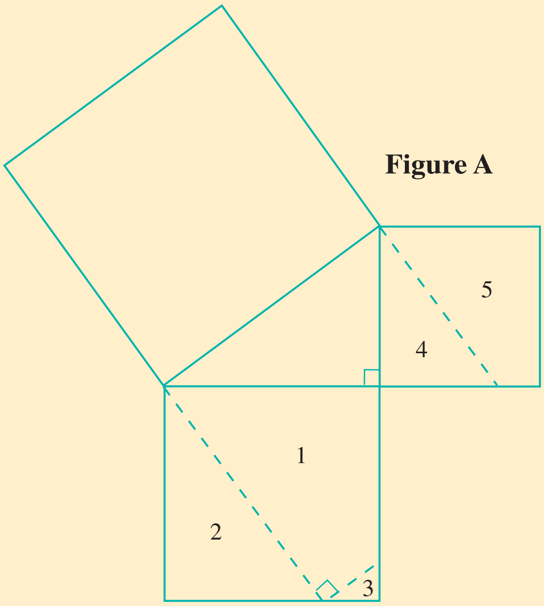


Figure A

Proof 2

Step 1: Make two copies of Figure B, on grid paper.

Step 2: Cut out the first copy of Figure B and paste it into your workbook, keeping all the squares intact.

Step 3: Cut out the second copy of Figure B. Then cut up the squares on the two shorter sides as shown. The pieces 2, 3, 4 and 5 are all the same size and shape as the triangle in the centre.

Step 4: Arrange the pieces to exactly cover the square on the hypotenuse. Paste this proof of Pythagoras' theorem into your book as well.

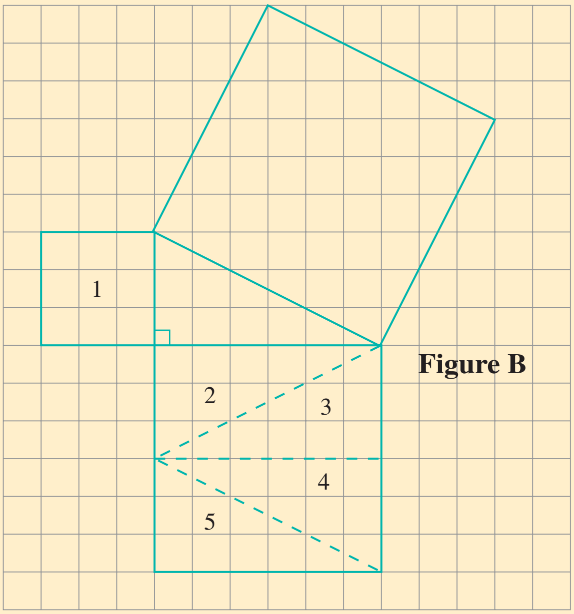


Figure B

Just for the record

Pythagoras and his secret society

Pythagoras' theorem is the most famous mathematical rule because it is simple, used widely and, according to the *Guinness Book of Records*, it is the rule that has been proved in the greatest number of different ways. A book published in 1940 contained 370 proofs, including one by US president James Garfield (1831–1881), and more have been discovered since.

Pythagoras was a Greek mathematician and philosopher who lived from 580 BC to 496 BC. He did not discover the theorem that we know by his name, as it was already known to ancient Babylonian, Hindu and Chinese mathematicians. However, Pythagoras may have been one of the first to prove the theorem.

In 540 BC, Pythagoras formed a school called the 'Pythagorean brotherhood', a secret society involved with philosophy, religion and mathematics. The Pythagoreans were vegetarians and believed in reincarnation, following a strict code of conduct and living in communes without possessions. The group included women, and all members were treated equally. The symbol of the brotherhood was a pentagram, a five-pointed star formed by the diagonals of a regular pentagon. Many rules attributed to Pythagoras were actually discovered by his followers.

The Pythagoreans believed that numbers had magical properties, that all objects were made up of numbers and that 'God is a mathematician'. The school motto was 'All is Number'. Pythagoras and his followers found mathematical patterns in music and in astronomy.

Draw a pentagram, the symbol of the 'Pythagorean brotherhood', using the diagonals of a regular pentagon.

Finding the length of the hypotenuse

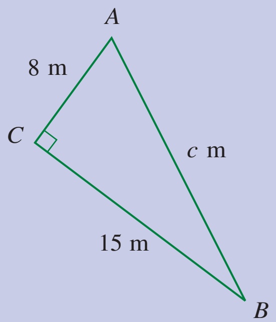
Pythagoras' theorem can be used to find the length of the hypotenuse of a right-angled triangle, given the lengths of the other two sides.

Skillsheet 6-01 Pythagoras' theorem

Spreadsheet 6-01 Pythagoras' theorem

Example 6

Find the length of the hypotenuse in ΔABC shown on the right.



Solution

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 8^2 + 15^2 \\ &= 64 + 225 \\ c^2 &= 289 \\ c &= \sqrt{289} \\ &= 17 \end{aligned}$$

$\therefore$ The length of the hypotenuse is 17 m.

Example 7

In ΔPQR, $\angle P = 90°$, $PQ = 25$ cm and $PR = 32$ cm. Sketch the triangle and find the length of the hypotenuse, p, correct to one decimal place.

Solution

$p^2 = q^2 + r^2$

$p^2 = 32^2 + 25^2$

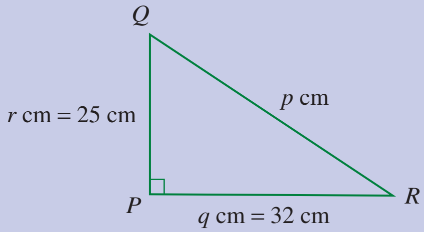
$p^2 = 1649$

$p = \sqrt{1649}$

$p = 40.607\,881\,01\ldots$

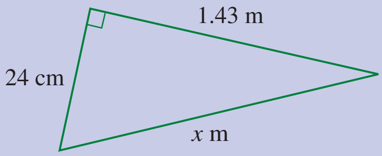
≈ 40.6

$\therefore$ The length of the hypotenuse is 40.6 cm (correct to one decimal place).



Example 8

Find x in the diagram on the right.



Solution

Since the length of x is required in metres, change 24 cm to 0.24 m.

$x^2 = 0.24^2 + 1.43^2$

$= 0.0576 + 2.0449$

$x^2 = 2.1025$

$x = \sqrt{2.1025}$

$= 1.45$

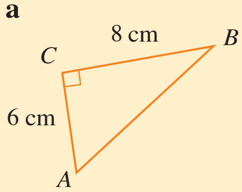
$\therefore$ The length of the hypotenuse is 1.45 m.

Exercise 6-04

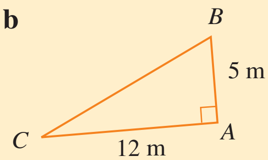
Example 6

1 Find the length of the hypotenuse in each of these triangles.

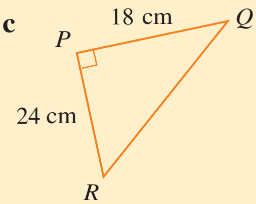
a



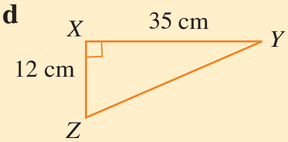
b



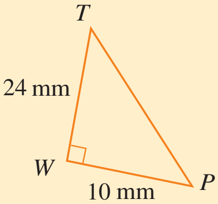
c



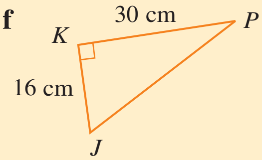
d



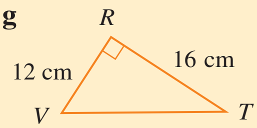
e



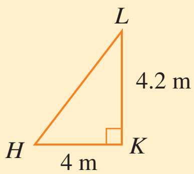
f



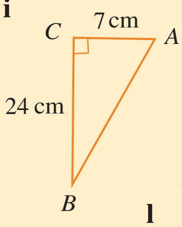
g



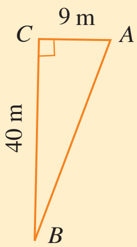
h



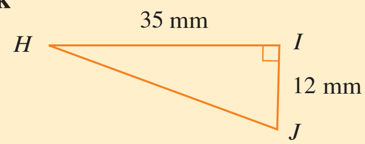
i



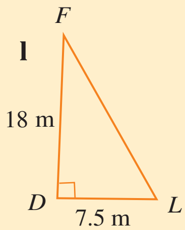
j



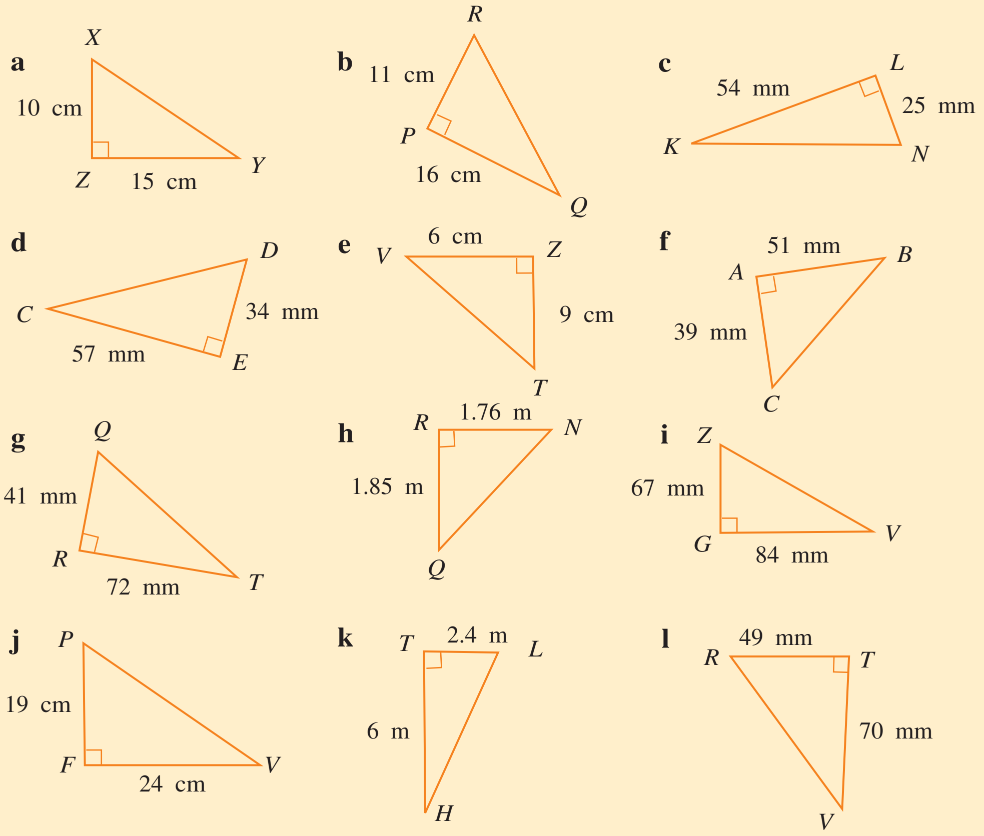
k



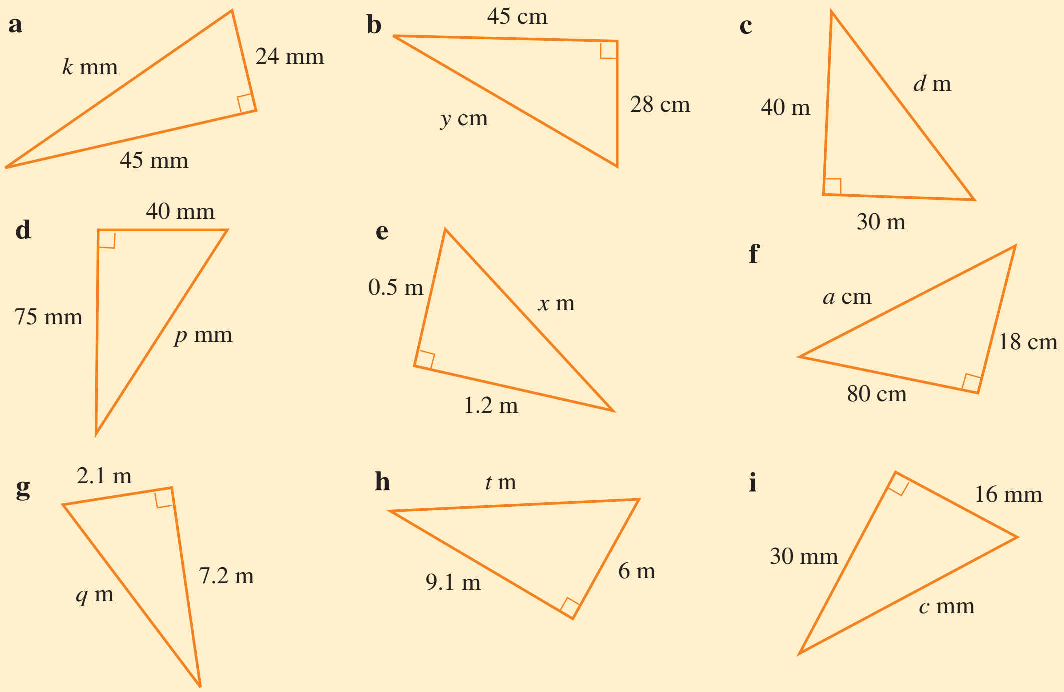
l



2 Find the length of the hypotenuse of each of these triangles, correct to one decimal place. **Example 8**

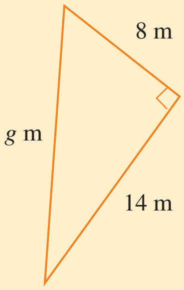


3 Find the value of the pronumeral in each of the following triangles (correct to one significant figure).

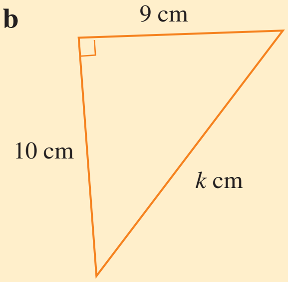


4 Find the value of each pronumeral, correct to one decimal place.

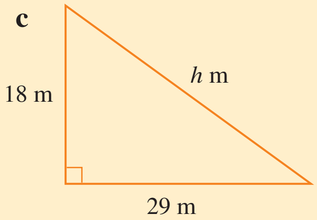
a



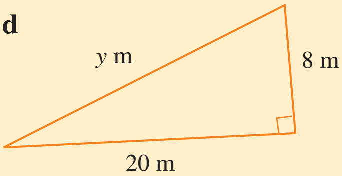
b



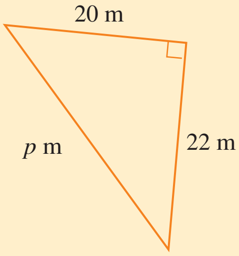
c



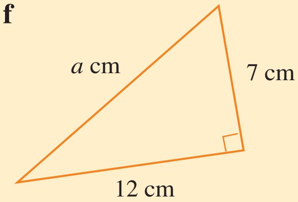
d



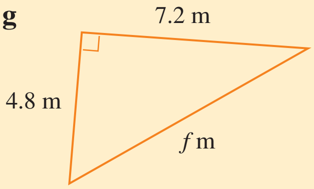
e



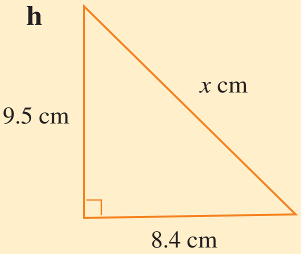
f



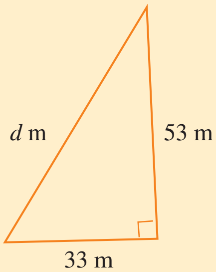
g



h



i



Example 7

5 For each of the following, sketch the triangle and then calculate the length of the hypotenuse. Make sure your answer is expressed in the correct units.

a In ΔABC, $\angle ABC = 90°$, $AB = 39$ cm, and $BC = 57$ cm. Find AC in centimetres, correct to one decimal place.

b In ΔMPQ, $\angle PQM = 90°$, $QM = 2.4$ m, and $PQ = 3.7$ m. Find PM in metres, correct to one decimal place.

c In ΔRVJ, $\angle J = 90°$, $JV = 127$ mm, $JR = 42$ mm. Find RV in millimetres, correct to the nearest millimetre.

d In ΔEGB, $EG = EB = 127$ mm, and $\angle GEB = 90°$. Find the length of BG in millimetres, correct to one decimal place.

e In ΔNKL, $\angle K = 90°$, $NK = 75$ cm, and $KL = 1.2$ m. Find NL in metres, correct to one decimal place.

f In ΔVZX, $\angle V = 90°$, $VX = 247$ mm, and $VZ = 30.6$ cm. Find ZX in millimetres, correct to the nearest millimetre.

g In ΔPQR, $\angle RPQ = 90°$, $PQ = 2.35$ m, and $PR = 2.2$ m. Find QR in metres, correct to two decimal places.

h In ΔDEF, $\angle F = 90°$, $DF = 84$ cm, and $EF = 1.45$ m. Find DE in centimetres, correct to the nearest whole number.

i In ΔPQR, $\angle QRP = 90°$, $PR = 11.6$ cm, and $RQ = 9.7$ cm. Find PQ in millimetres, correct to the nearest whole number.

j In ΔYTM, $\angle T = 90°$, $TM = 3.4$ m, and $TY = 2.75$ m. Find YM correct to three decimal places.

Using technology

Spreadsheet activity: Finding the hypotenuse

Spreadsheet

1 Set up your spreadsheet as follows:

	A	B	C
1	a	b	c
2			= sqrt(A2^2 + B2^2)
3			
4			
⋮			

- The labels a, b, and c represent the length of the sides of a right-angled triangle, with c being the hypotenuse.
- To obtain values for c in rows 3, 4, …, fill down from C2.

2 Use your spreadsheet to find the length of the hypotenuse given the following pairs of values for the other two sides:

a $a = 5, b = 12$ **b** $a = 8, b = 8$ **c** $a = 12, b = 7$
d $a = 46, b = 22$ **e** $a = 30, b = 25$ **f** $a = 18, b = 12$
g $a = 80, b = 18$ **h** $a = 8, b = 4$ **i** $a = 10, b = 1$

3 Print your results and paste them into your workbook.

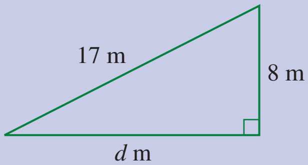
Finding the length of a shorter side

Skillsheet 6-01 Pythagoras' theorem

Pythagoras' theorem can also be used to find the length of a shorter side of a right-angled triangle.

Example 9

Find the value of the pronumeral in the diagram on the right.



Solution

$17^2 = d^2 + 8^2$

$\therefore d^2 + 8^2 = 17^2$ (placing d^2 on the left-hand side)

$d^2 = 17^2 - 8^2$ (subtracting 8^2 from both sides)

$= 289 - 64$

$d^2 = 225$

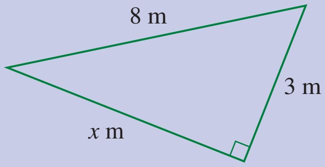
$d = \sqrt{225}$

$= 15$

$\therefore$ The value of d is 15.

Example 10

Find x (correct to one decimal place).



Solution

$$
\begin{aligned}
8^2 &= x^2 + 3^2 \\
x^2 + 3^2 &= 8^2 \\
x^2 &= 8^2 - 3^2 \\
&= 64 - 9 \\
x^2 &= 55 \\
x &= \sqrt{55} \\
&= 7.41619\ldots \\
x &\approx 7.4
\end{aligned}
$$

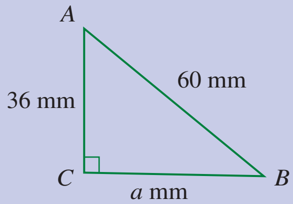
$\therefore$ The length of x is 7.4 m.

Example 11

In ΔABC, $\angle C = 90°$, $AC = 36$ mm, and $AB = 60$ mm. Find the length of BC.

Solution

Sketch ΔABC from the given information.



$c^2 = a^2 + b^2$ (Pythagoras' theorem)

$60^2 = a^2 + 36^2$ (substituting given information)

$a^2 + 36^2 = 60^2$ (placing a^2 on the left-hand side)

$\therefore a^2 = 60^2 - 36^2$ (subtracting 36^2 from both sides)

$$
\begin{aligned}
a^2 &= 3600 - 1296 \\
a^2 &= 2304 \\
a &= \sqrt{2304} \\
&= 48
\end{aligned}
$$

$\therefore$ The length of BC is 48 mm.

Exercise 6-05

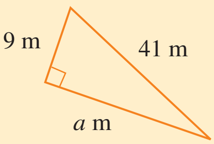
Example 9

SkillBuilder 21-16 Pythagoras' theorem

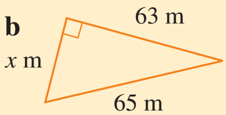
Worksheet 6-05 Finding a missing side

1 Find the value of each pronumeral:

a



b



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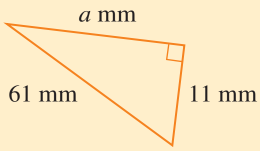

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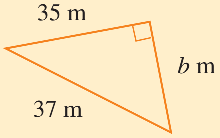

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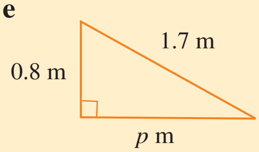

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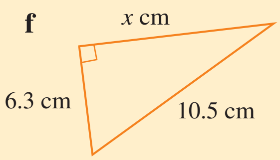

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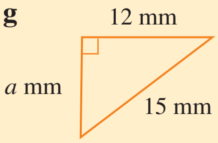

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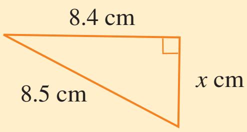

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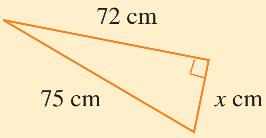

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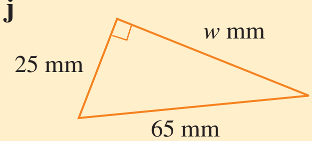

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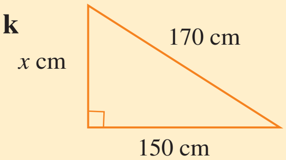

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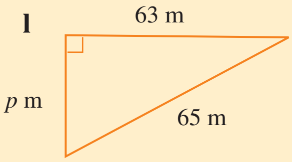
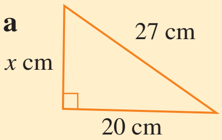


2 Find the length of the unknown side in each of these triangles. Give your answers correct to one decimal place.

Example 10

a



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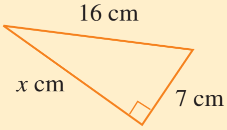

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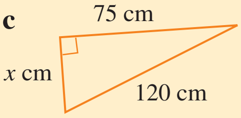

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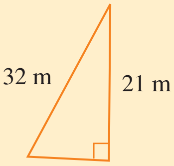

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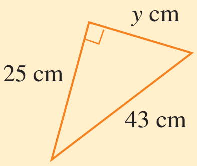

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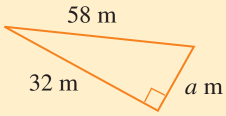

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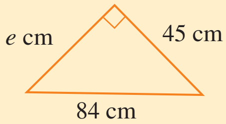

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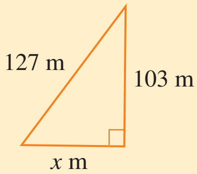

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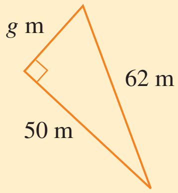

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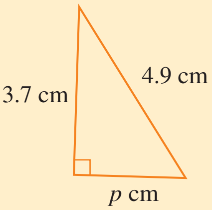

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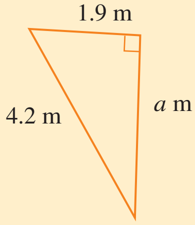

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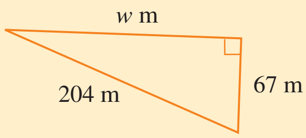
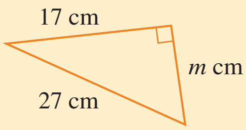


3 Find the length of the unknown side of each of these triangles. Give your answers correct to one decimal place.

Example 11

a



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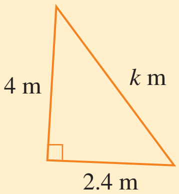

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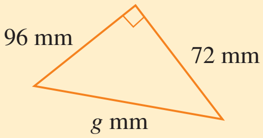

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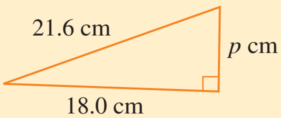

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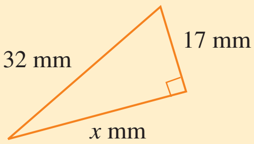

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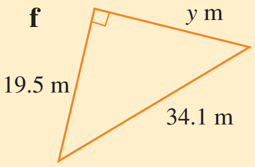

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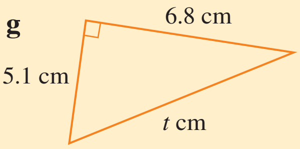

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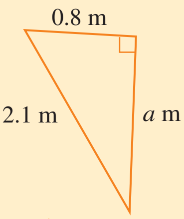

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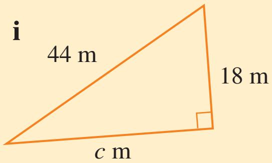

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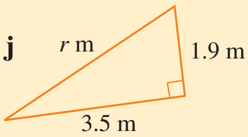

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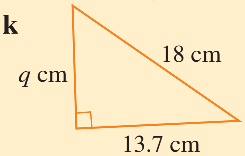

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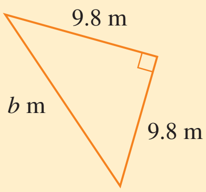


4 For each of the following, sketch the triangle and then calculate the length of the unknown side. Make sure your answer is expressed in the correct units.

a In ΔABC, $\angle C = 90°$, $CA = 57$ mm, and $AB = 80$ mm. Find BC correct to the nearest millimetre.

b In ΔPQR, $\angle RPQ = 90°$, $PQ = 20$ cm, and $QR = 55$ cm. Calculate RP correct to one decimal place.

c In ΔFHJ, $\angle JHF = 90°$, $HJ = 1.3$ m, and $FJ = 1.65$ m. Find FH in metres, correct to one decimal place.

d In ΔXYZ, $\angle X = 90°$, $YZ = 2.4$ m, and $XZ = 1.3$ m. Calculate XY in centimetres, correct to the nearest centimetre.

e In ΔPTV, $\angle P = 90°$, $TV = 230$ cm, and $PV = 67$ cm. Find PT in centimetres, correct to one decimal place.

f In ΔGKL, $\angle LGK = 90°$, $KL = 527$ mm, and $GK = 21.6$ cm. Find GL in millimetres, correct to the nearest whole number.

g In ΔZXY, $\angle X = 90°$, $ZY = 1.07$ m, and $XY = 234$ mm. Find XZ in millimetres, correct to one decimal place.

h In ΔSPV, $\angle V = 90°$, $PS = 190$ cm, and $PV = 84$ mm. Find SV in centimetres, correct to one decimal place.

Using technology

Pythagoras' theorem

You can solve problems involving Pythagoras' theorem by using a graphics calculator. The formula for Pythagoras' theorem is written as $H^2 = A^2 + B^2$, where H is the hypotenuse, and A and B are the other two sides. The formula needs to be arranged to the form $0 = A^2 + B^2 - H^2$.

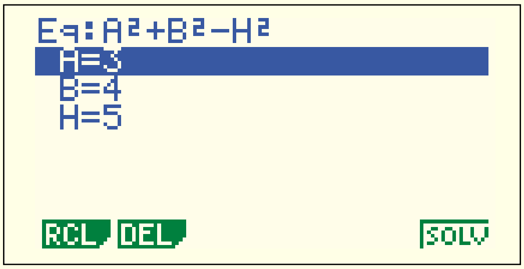
Casio CFS-9850GB PLUS

Step 1: Move to EQUA on the main menu and press EXE. Select F3 Solver.

Step 2: Next to 'Eq:' type $A^2 + B^2 - H^2$.

Step 3: Type in the values of the variables you know and type 0 for the value you don't know.

Step 4: Press F5 to work out the unknown.



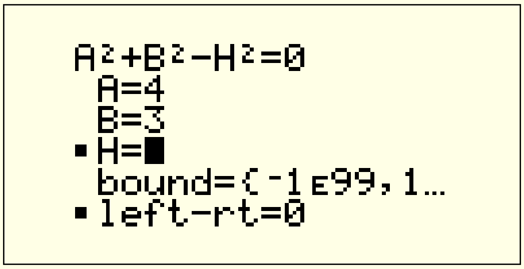
Texas Instruments TI-83

Step 1: Press MATH 0 (Solver).

Step 2: Next to 'eqn: 0 =' type $A^2 + B^2 - H^2$, and press ENTER. You may need to use the 'up' arrow to show the equation. The screen will show the variables.

Step 3: Type in the values you know and type 0 for the value you don't know.

Step 4: Press ALPHA ENTER (SOLVE) to work out the unknown.



1 Use your calculator to answer the questions in Exercise 7-05.

Using technology

Spreadsheet activity: Finding the length of a shorter side

For a right-angled triangle with sides, a, b, and c (where c is the hypotenuse), a shorter side can be found by using the formula:

$$b = \sqrt{c^2 - a^2}$$

Spreadsheet

1 Set up your spreadsheet as shown below:

	A	B	C
1	c	a	b
2			= sqrt(A2^2 – B2^2)
3			
⋮			

2 Use your spreadsheet to find the length of one of the shorter sides in a right-angled triangle given the following pairs of values for the other two sides:

a $a = 7, c = 15$ **b** $a = 6.8, c = 10.9$ **c** $a = 84, c = 100$
d $a = 2, c = 4$ **e** $a = 22, c = 40$ **f** $a = 0.8, c = 1.8$
g $a = 5.7, c = 15.8$ **h** $a = 9.5, c = 15.5$

3 Print your results and paste them into your workbook.

Working mathematically

Reasoning: Pythagorean triads

A **Pythagorean triad** (or **triple**) is a group of three numbers that obey Pythagoras' rule (the square of the largest number is equal to the sum of the squares of the other two numbers).

Consider the numbers, 5, 12 and 13. The number 13 is the largest number.

$$13^2 = 169$$

$$5^2 + 12^2 = 25 + 144$$
$$= 169$$

So, $5^2 + 12^2 = 13^2$, and this means that (5, 12, 13) is a Pythagorean triad.

1 Show that each of the following sets of numbers is a Pythagorean triad.

a (11, 60, 61) **b** (8, 15, 17)
c (72, 96, 120) **d** (7, 24, 25)

2 Which of the following sets of numbers are Pythagorean triads? Check your answers with those of other students.

a (36, 48, 60) **b** (13, 60, 61) **c** (21, 72, 75)
d (9, 40, 43) **e** (9, 12, 15) **f** (9, 40, 41)

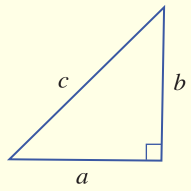
3 a The group (3, 4, 5) is a Pythagorean triad. Are multiples of (3, 4, 5) also Pythagorean triads? Explain your answer.

b Use the Pythagorean triad (5, 12, 13) to form three other triads. Compare your results with those of other students.

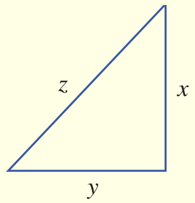
Testing for right-angled triangles

Worksheet 6-06
Pythagorean practice

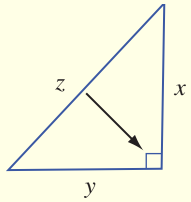
If a triangle with sides of length a, b, and c, as shown, is right-angled, then we know, by Pythagoras' theorem, that $c^2 = a^2 + b^2$.



Conversely, for a triangle with sides of length x, y and z, as shown, if $x^2 + y^2 = z^2$ then we know that the triangle must be right-angled. (The right angle will be opposite the hypotenuse, z.)

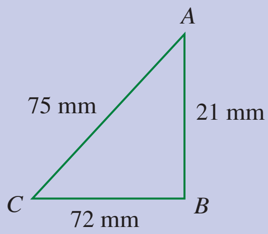


Expressing this rule another way, if the side lengths of a triangle form a Pythagorean triad, then the triangle must be right-angled.



Example 12

1 Is the triangle shown on the right a right-angled triangle?



Solution

$$\begin{aligned} 21^2 + 72^2 &= 441 + 5184 \\ &= 5625 \\ 75^2 &= 5625 \end{aligned}$$

Since $21^2 + 72^2 = 75^2$, the triangle is right-angled.
(The right angle is $\angle B$).

2 Dan was asked to construct a right-angled triangle. The dimensions of his triangle were 37 cm, 12 cm and 40 cm. How do you know these measurements do *not* form a right-angled triangle?

Solution

If Dan's triangle is right-angled, then $12^2 + 37^2 = 40^2$.

But: $12^2 + 37^2 = 144 + 1369$
$\quad = 1513$

and: $40^2 = 1600$

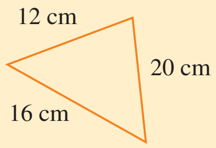
Since $12^2 + 37^2 \neq 40^2$, the triangle is *not* right angled.

Exercise 6-06

Example 12

1 From the measurements shown, test which of the following triangles are right angled. For each right-angled triangle, copy the triangle and mark the position of the right angle.

a



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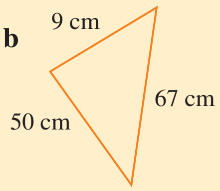

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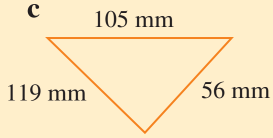
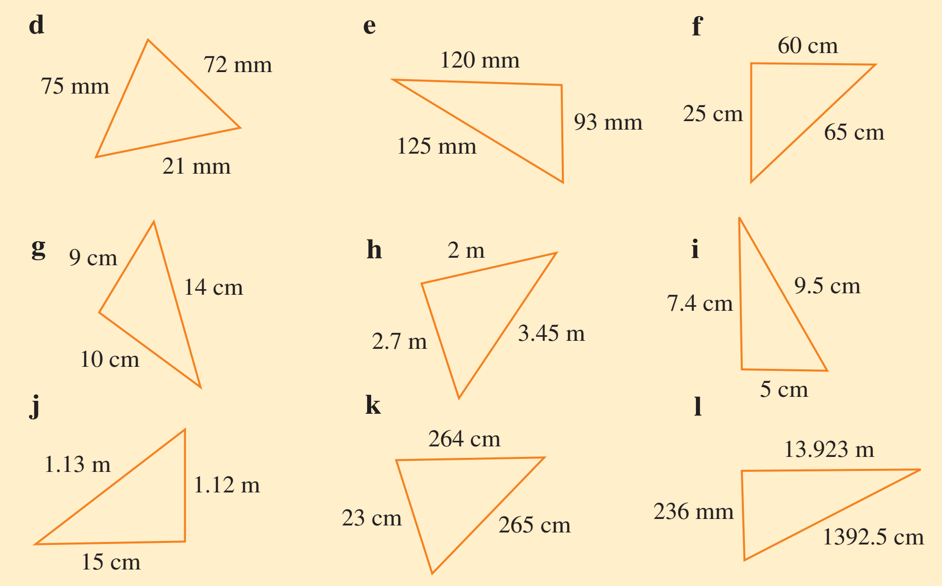


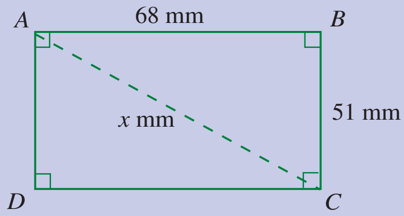


Pythagoras' theorem in quadrilaterals

Worksheet 6-06 Pythagorean practice

Example 13

1 How long is the diagonal AC in this rectangle?



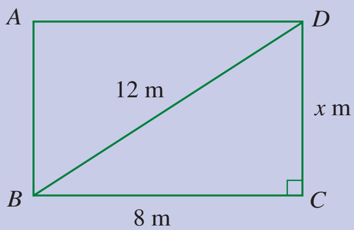
Solution

Let x be the length of the diagonal.

$$\begin{aligned} x^2 &= 68^2 + 51^2 \\ &= 4624 + 2601 \\ x^2 &= 7225 \\ x &= \sqrt{7225} \\ &= 85 \end{aligned}$$

$\therefore$ The diagonal, AC, is 85 mm long.

2 Find the width, CD of this rectangle (correct to one decimal place).



Solution

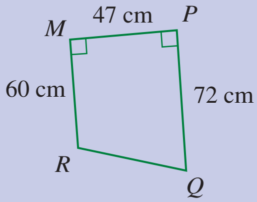
Let CD be x m.

$$\begin{aligned} 12^2 &= x^2 + 8^2 \\ x^2 + 8^2 &= 12^2 \\ x^2 &= 12^2 - 8^2 \\ &= 144 - 64 \\ x^2 &= 80 \\ x &= \sqrt{80} \\ &= 8.944\ 27 \ldots \\ &\approx 8.9 \end{aligned}$$

$\therefore$ The width, CD, of the rectangle is 8.9 m.

Example 14

Find the length of RQ in this trapezium, correct to one decimal place.



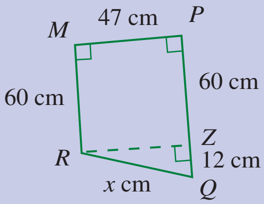
Solution

Form the right-angled triangle RZQ, as shown below, and let RQ be x cm.

By comparing sides:

$ZQ = 72 - 60 = 12$ cm

$RZ = MP = 47$ cm

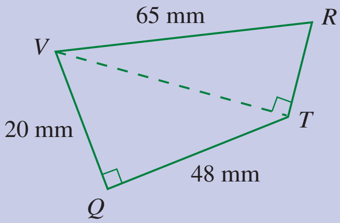


$$\therefore x^2 = 47^2 + 12^2$$
$$= 2209 + 144$$
$$x^2 = 2353$$
$$x = \sqrt{2353}$$
$$= 48.507\,731\,34\ldots$$
$$\approx 48.5$$

$\therefore$ The length of RQ is 48.5 cm (correct to one decimal place).

Example 15

Find the length of RT in this quadrilateral:



Solution

Find the length of VT first and then use it to find the length of RT .

Let VT be q mm.

$q^2 = 20^2 + 48^2$ (using Pythagoras' theorem in ΔVTQ)

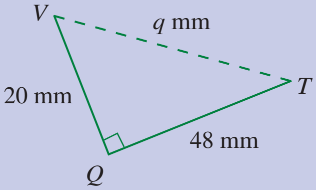
$= 400 + 2304$

$q^2 = 2704$

$q = \sqrt{2704}$

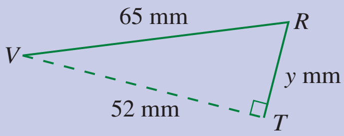
$= 52$

$\therefore$ The length of VT is 52 mm.



Now use $VT = 52$ mm and Pythagoras' theorem in ΔVTR. Let RT be y mm.

$$65^2 = 52^2 + y^2$$
$$4225 = 2704 + y^2$$
$$4225 - 2704 = y^2$$
$$1521 = y^2$$
$$y = \sqrt{1521}$$
$$= 39$$



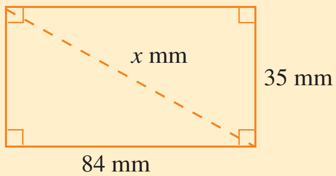
$\therefore$ The length of RT is 39 mm.

Exercise 6-07

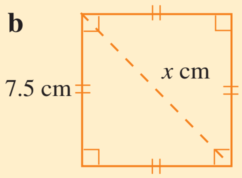
Example 13

1 Find the size of the pronumeral in each of these quadrilaterals. Give your answer correct to two decimal places where necessary.

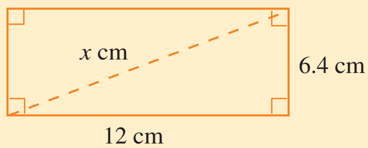
a



b



c



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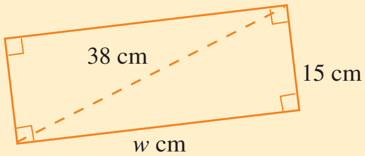
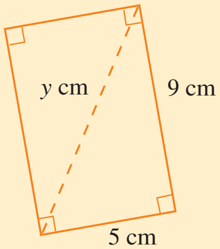
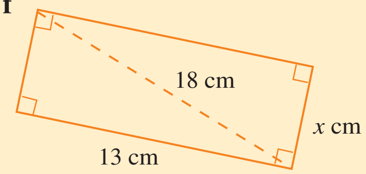


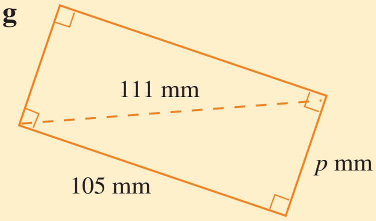
e



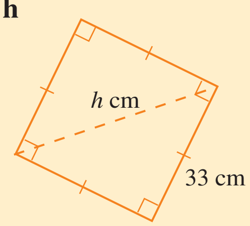
f



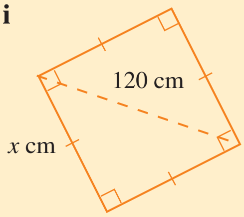
g



h



i



2 a A rectangle, *PQRT*, has side lengths of 75 mm and 100 mm. Find the length of the diagonal *PR*.

b A square has a side length of 96 mm. Find the length of its diagonals, correct to the nearest millimetre.

c A rectangle, *ZVXY*, has side lengths of 184 mm and 230 mm. Find the length of the diagonal *ZX* (correct to the nearest millimetre).

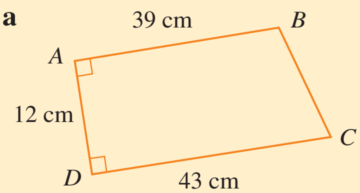
d The diagonal of a rectangle is 200 mm in length. One of the side lengths of the rectangle is 167 mm. Find the length of the other side, correct to the nearest millimetre.

e A 40 cm piece of wire is bent to form a square. How long is the diagonal of the square (correct to the nearest centimetre)?

3 Find the length of each unknown side of these quadrilaterals. Give your answer correct to one decimal place where necessary.

Example 14

a



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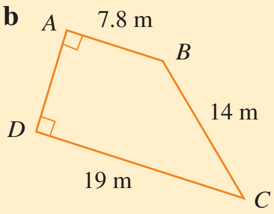

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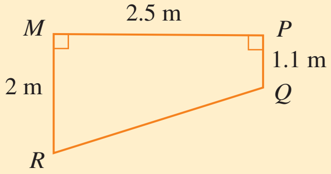
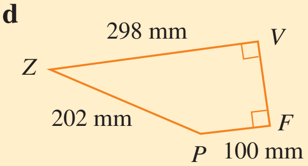
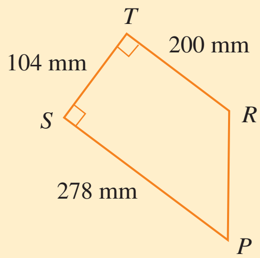


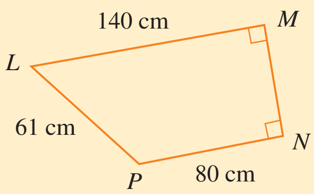
d



e



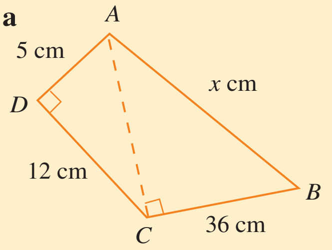
f



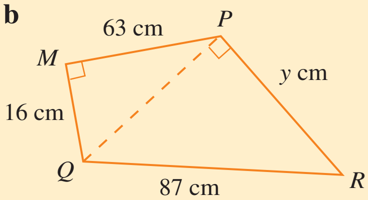
Example 15

4 Find the value (in centimetres) of each pronumeral in the following quadrilaterals. Give your answer correct to one decimal place where necessary.

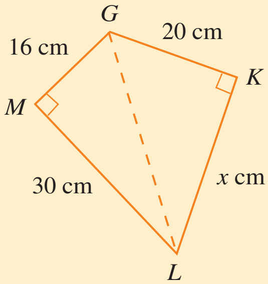
a



b



c



Skillbank 6

SkillTest 6-01

Estimating square roots

Estimating square roots

The square numbers can be used to estimate square roots.

Number	1	2	3	4	5	6	7	8	9	10	11	12
Square	1	4	9	16	25	36	49	64	81	100	121	144

1 Examine these examples:

a Estimate $\sqrt{7}$.

7 is between the square numbers 4 and 9 (2^2 and 3^2 respectively).

Now, because 7 is closer to 3^2 than to 2^2, $\sqrt{7}$ will be closer to 3 than to 2.

An estimate is $\sqrt{7} \approx 2.6$. (Actual answer = 2.645 …)

b Estimate $\sqrt{55}$.

55 is between the square numbers 49 and 64 (7^2 and 8^2 respectively).

55 is closer to 7^2 than to 8^2, $\sqrt{55}$ will be closer to 7 than to 8.

An estimate is $\sqrt{55} \approx 7.4$. (Actual answer = 7.416 …)

2 Now estimate these square roots:

a $\sqrt{8}$ **b** $\sqrt{18}$ **c** $\sqrt{28}$ **d** $\sqrt{111}$

e $\sqrt{80}$ **f** $\sqrt{31}$ **g** $\sqrt{12}$ **h** $\sqrt{65}$

i $\sqrt{75}$ **j** $\sqrt{29}$ **k** $\sqrt{40}$ **l** $\sqrt{126}$

Practical applications of Pythagoras' theorem

When using Pythagoras' theorem to solve problems, it is often useful to follow the steps below.

- Read the problem carefully.
- Draw a diagram and label any given information. Choose a variable to represent the length you want to find.
- If needed, make a separate sketch of the right-angled triangle you will use for calculations.
- Use Pythagoras' theorem to calculate the length.
- Answer the question.

Example 16

1 A wire supporting a tower is 20 m in length. It is attached to the ground 10 m from the base of the tower. Find how far the wire reaches up the tower, giving your answer to the nearest 0.1 m.

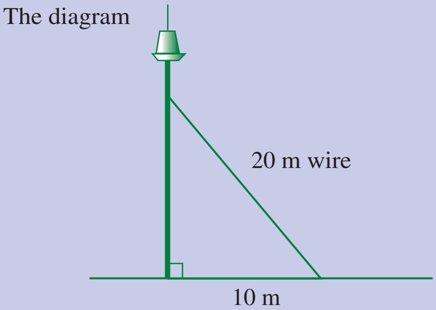
Solution

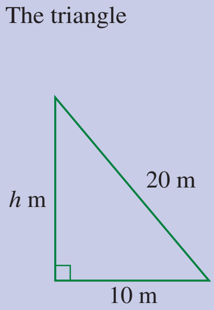
First use the given information to draw the diagram and the triangle.

Let h represent how far the wire reaches up the tower.

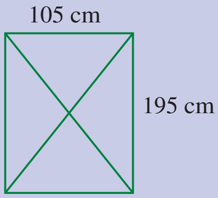
$$\begin{aligned} 20^2 &= 10^2 + h^2 \\ 400 &= 100 + h^2 \\ 400 - 100 &= h^2 \\ 300 &= h^2 \\ h &= \sqrt{300} \\ &= 17.3205 \\ &\approx 17.3 \end{aligned}$$

$\therefore$ The wire reaches 17.3 m up the tower.





2 A screen door is reinforced by two metal braces crossing the door diagonally. Find (in metres) the length of metal used in the two braces on the door.

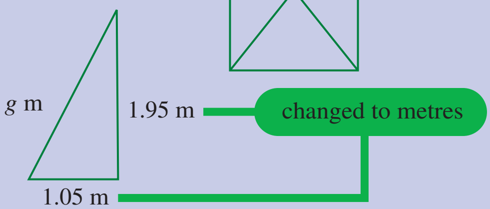


Solution

Let g represent the length of a brace.

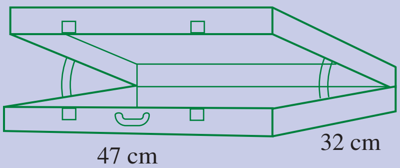
$$\begin{aligned} g^2 &= 1.95^2 + 1.05^2 \\ g^2 &= 4.905 \\ g &= \sqrt{4.905} \\ &\approx 2.21 \text{ (correct to two decimal places)} \end{aligned}$$

$\therefore$ The length of metal required for two braces is 2×2.21 m $= 4.42$ m.



(*Note*: The two braces are the same length because the diagonals of a rectangle are equal.)

3 Ms Turco's briefcase has a rectangular base of 47 cm × 32 cm. Can her 60 cm umbrella fit in the briefcase, assuming that it must lie flat on the base?



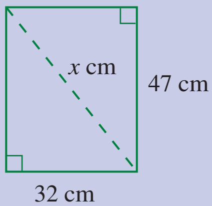
Solution

If the diagonal of the base of the briefcase is greater than or equal to 60 cm, then Ms Turco's umbrella will fit in it.

Call the length of the diagonal x.

$$\begin{aligned} x^2 &= 47^2 + 32^2 \\ x^2 &= 3233 \\ x &= \sqrt{3233} \\ &= 56.8594 \ldots \end{aligned}$$

$\therefore x < 60$



Since the diagonal of the base of the briefcase is less than the length of the umbrella, the umbrella will not fit into the briefcase.

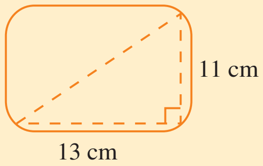
Exercise 6-08

Example 16

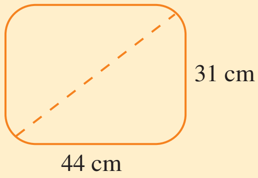
SkillBuilder 21-11 Using Pythagoras' theorem

1 The size of a television screen is described by the length of its diagonal. So a 48 cm set has a diagonal length of 48 cm. Find the size of each of these TV screens, giving your answer correct to the nearest centimetre.

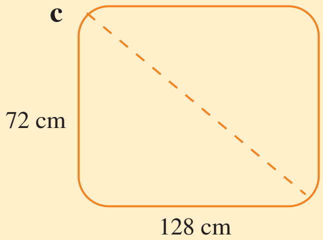
a



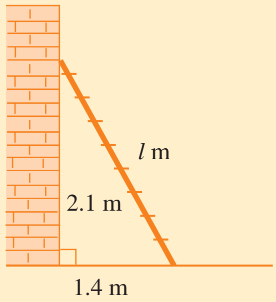
b



c



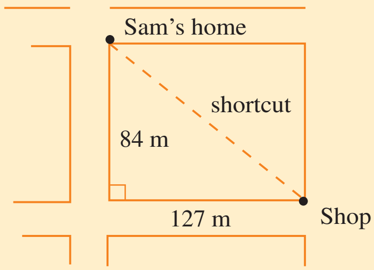
2 Find the length (l) of a ladder which reaches 2.1 m up a wall when the foot of the ladder is 1.4 m from the bottom of the wall. Give your answer to the nearest centimetre.



3 Sam takes a shortcut from his home to the shop. Find:

a the length (to the nearest metre) of the shortcut

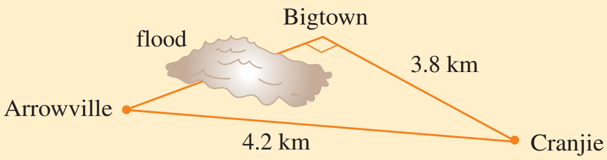
b the distance Sam saves by not going the long way to the shop.



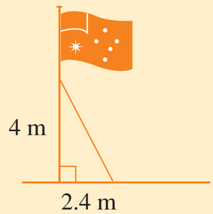
4 The road from Arrowville to Bigtown is cut by floodwater, so Alison must travel from Arrowville to Bigtown via Cranjie. Find:

a the distance (correct to one decimal place) from Arrowville direct to Bigtown

b the extra distance Alison travels by taking the detour via Cranjie.



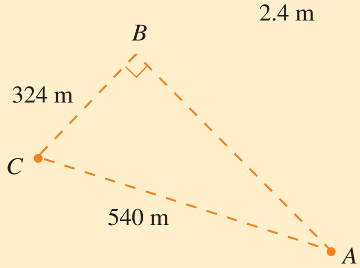
5 A flagpole is supported by a piece of wire which reaches 4 m up the flagpole. The wire is attached to the ground 2.4 m from the base of the flagpole. Find the length of the wire, correct to one decimal place.



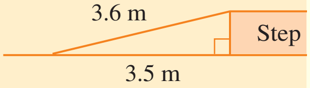
6 On a cross-country ski course, checkpoints C and A are 540 m apart and checkpoints C and B are 324 m apart. Find:

a the distance between checkpoints A and B

b the distance covered in one lap of the course, giving your answer in kilometres, correct to one decimal place.

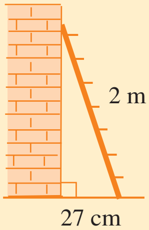


7 The end of a 3.6 m plank touches the ground 3.5 metres from the bottom of a step as shown. How high (to the nearest centimetre) is the step?

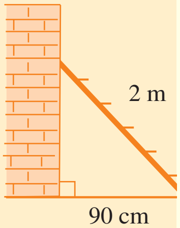


8 For safety, a 2 m ladder should be placed no less than 27 cm, and no more than 90 cm, from a wall. Calculate, to the nearest centimetre:

- **a** the maximum height on the wall reached by the ladder
- **b** the minimum height on the wall reached by the ladder



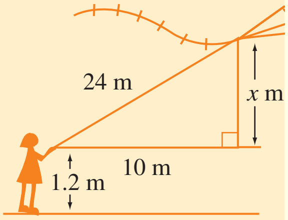
Minimum distance from wall



Maximum distan from wall

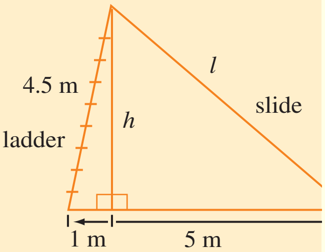
9 A kite is attached to a 24 m piece of rope, as shown. The rope is held 1.2 m above the ground and covers a horizontal distance of 10 m. Find:

- **a** x, correct to one decimal place
- **b** the height of the kite above the ground, correct to the nearest metre.



10 A playground slide is made up of two right-angled triangles. Giving your answer to the nearest millimetre, find:

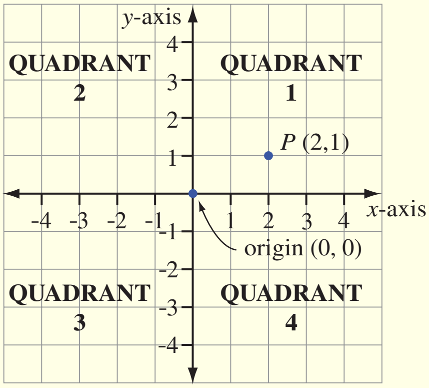
- **a** the height (h) of the slide
- **b** the length (l) of the slide.



Intervals on the number plane

Some terms and symbols

The diagram below shows a **Cartesian plane** (or **number plane**).




The Cartesian number plane was named after René Descartes (1596–1650).

Points can be represented by an ordered pair of coordinates (x, y). For example, the point P has coordinates (2, 1).

Example 17

Plot these points on a number plane: $A(5, 3)$ $B(-4, -3)$ $C(-2, 5)$ $D(5, -5)$

a Draw the interval AD.

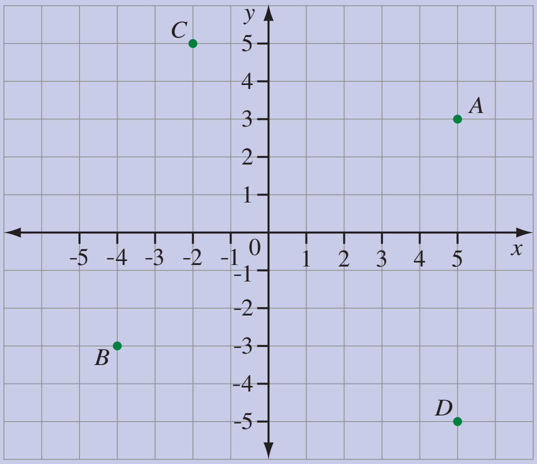
b What is the length of the interval AD?

Solution

To plot $A(5, 3)$ go across 5 to the **right** from the origin and then **up** 3.

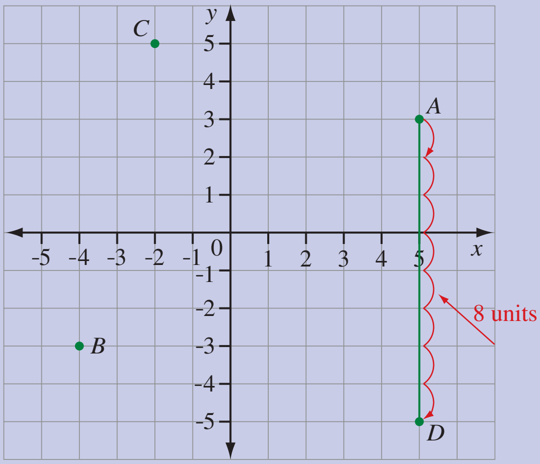
To plot $B(-4, -3)$, go across 4 to the **left** from the origin and then **down** 3.

For C we go **left** 2 from the origin and **up** 5. For D we go **right** 5 from the origin and **down** 5.



a Draw a line from A to D. This is the interval AD.

b The length of AD is 8 units.



Using intervals to form right-angled triangles

Two points can be graphed to form the endpoints of an interval. If the interval is neither horizontal nor vertical, a right-angled triangle can be formed with that interval as the hypotenuse.

To do this:

- draw a vertical side from the higher point, and
- draw a horizontal side from the lower point.

Example 18

Plot the points $A(-3, 2)$ and $B(5, -4)$ on a number plane.

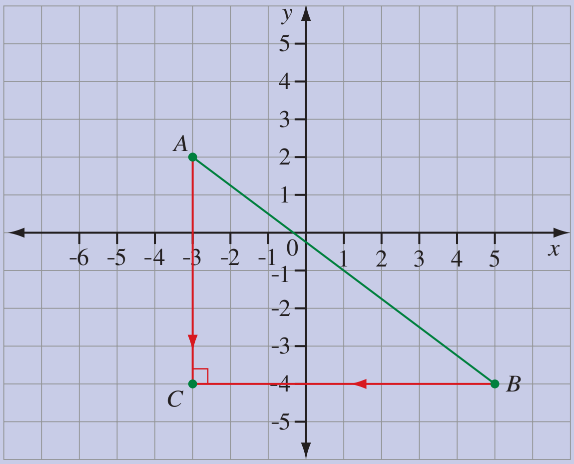
a With the interval AB as the hypotenuse, form a right-angled triangle.

b What are the lengths of the horizontal and vertical sides of the triangle?

Solution

a A vertical side is drawn down from A (the higher point) and a horizontal side is drawn to the left from B (the lower point). The two sides meet at C.

b The horizontal side BC is 8 units long. The vertical side AC is 6 units long.



Exercise 6-09

Example 17

Worksheet Appendix 4
Number plane grid paper

1 For each of the following pairs of points:

i plot the pair of points on a number plane **ii** draw the interval joining the points.

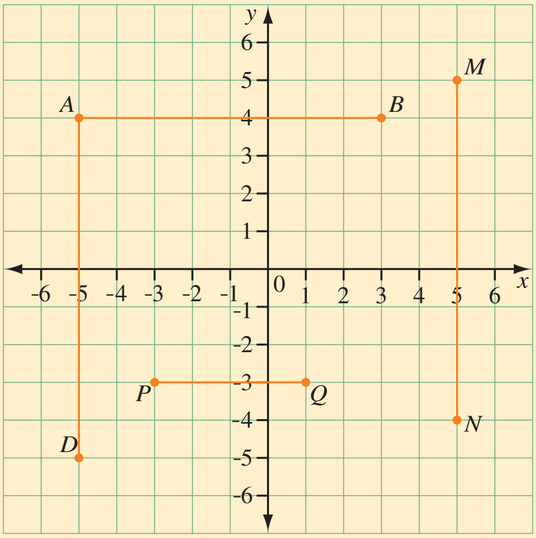
a $A(-3, 2)$ and $B(5, -2)$ **b** $C(1, 3)$ and $D(-2, -1)$

c $E(0, 5)$ and $F(3, -2)$ **d** $G(-3, 1)$ and $H(2, 4)$

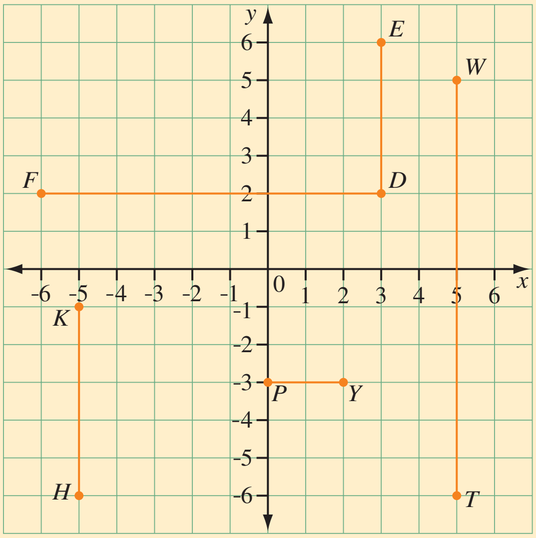
e $J(-2, -4)$ and $K(3, 0)$ **f** $L(4, 3)$ and $M(-3, -1)$

2 Find the lengths of all the intervals shown on each of these number planes.

a



b



Example 18

3 Plot the points $T(2, -4)$ and $W(-1, 2)$.

a With the interval TW as the hypotenuse, form a right-angled triangle.

b What are the lengths of the horizontal and vertical sides of the triangle?

4 For each of the following pairs of points:
- **i** plot the pair of points on a number plane
- **ii** draw the interval joining the two points
- **iii** with the interval as the hypotenuse, form a right-angled triangle
- **iv** find the lengths of the horizontal and vertical sides of the triangle.

a $B(-3, 0)$ and $F(3, 4)$ **b** $K(6, 3)$ and $L(-4, -1)$ **c** $P(2, -3)$ and $N(-4, 5)$
d $H(-3, -5)$ and $O(0, 0)$ **e** $W(0, 4)$ and $E(-3, -2)$ **f** $G(3, 1)$ and $Q(-4, -4)$
g $J(-3, -6)$ and $R(3, 4)$ **h** $A(1, -2)$ and $D(-1, 5)$ **i** $M(-5, 0)$ and $R(0, -4)$

Finding the length of an interval

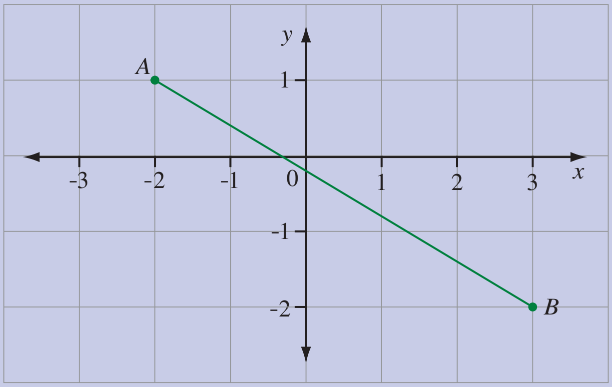
Worksheet 6-07
Length of an interval

Spreadsheet 6-02
Interval length

Pythagoras' theorem may be used to find the length of an interval on the Cartesian plane.

Example 19

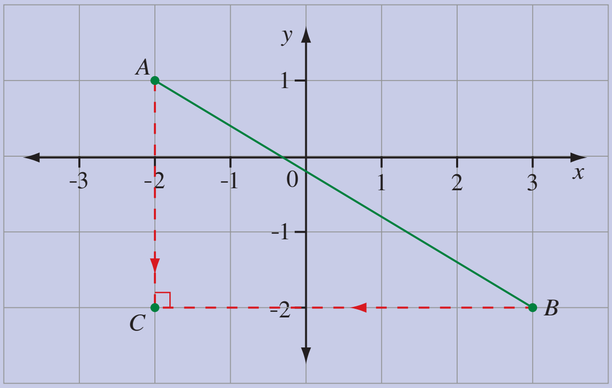
The points $A(-2, 1)$ and $B(3, -2)$ are shown on the number plane on the right. Find, correct to one decimal place, the length of the interval AB.



Solution

Form a right-angled triangle ABC by drawing a vertical side from A (the higher point), and a horizontal side from B (the lower point) as shown.

$\therefore$ The point C is $(-2, -2)$.
So $AC = 3$ units, and $BC = 5$ units.



Using Pythagoras' theorem:

$$AB^2 = 3^2 + 5^2$$
$$= 9 + 25$$
$$AB^2 = 34$$
$$\therefore AB = \sqrt{34}$$
$$\approx 5.8 \text{ (correct to one decimal place).}$$

Note: The point C could also be $(3, 1)$. Can you explain why?

Exact length

The answer of 5.8 units in Example 19 represents the approximate length of AB, correct to one decimal place. When the length was expressed using the irrational number $\sqrt{34}$, it was the **exact** length of AB.

Example 20

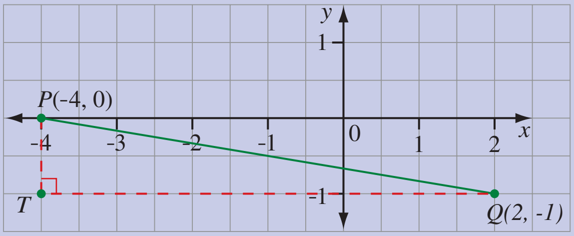
Find the length of the interval joining $P(-4, 0)$ to $Q(2, -1)$, expressing the answer in exact form.

Solution

First plot P and Q on a number plane and find another point (call it T) such that ΔPQT is right-angled at T. [Note that T could also be (2, 0) to make the right-angled triangle PTQ. It makes no difference to the length PQ].

So $PT = 1$ unit, and $TQ = 6$ units.

$$PQ^2 = 6^2 + 1^2$$
$$= 36 + 1$$
$$PQ^2 = 37$$
$$PQ = \sqrt{37} \text{ (exact length)}$$



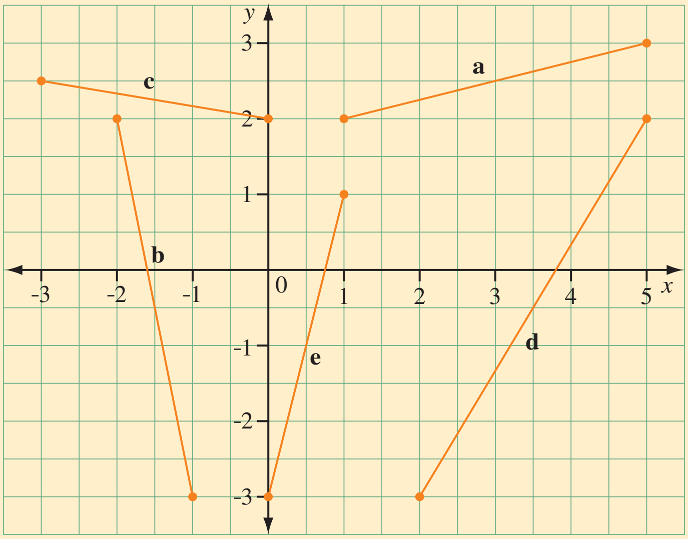
Exercise 6-10

Example 19

1 For each interval shown on the number plane on the right, use Pythagoras' theorem to find:

i the exact length of the interval

ii its length correct to one decimal place.



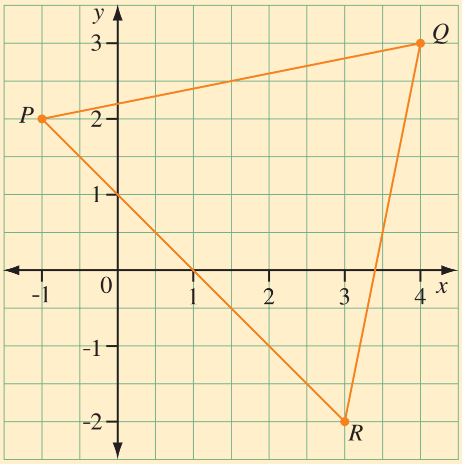
2 Find the distance between each pair of points A and B by plotting them on a Cartesian plane. Give your answer correct to one decimal place where necessary.

a $A(3, 2), B(6, -2)$	**b** $A(-1, -1), B(4, 3)$	**c** $A(-2, 5), B(1, 2)$
d $A(3, 0), B(4, 6)$	**e** $A(-5, 2), B(0, -1)$	**f** $A(6, -1), B(-3, 2)$
g $A(-2, -2), B(3, 2)$	**h** $A(5, -4), B(0, 1)$	**i** $A(3, 5), B(2, 4)$
j $A(-1, 4), B(5, -4)$	**k** $A(0, -5), B(5, 0)$	**l** $A(4, -3), B(-4, 3)$

Worksheet Appendix 4
Number plane grid paper

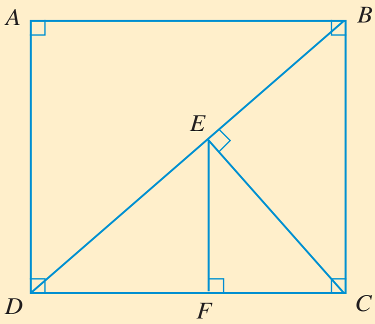
Example 20

3 **a** Find the exact length of PQ in the diagram on the right.
b Find the exact length of QR.
c Find the exact length of PR.
d What type of triangle is ΔPQR?
e Find the perimeter of ΔPQR to one decimal place.



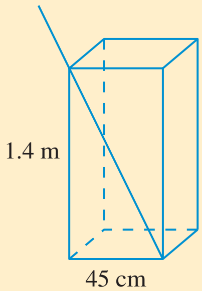
Power plus

1 Name five hypotenuses in this diagram:



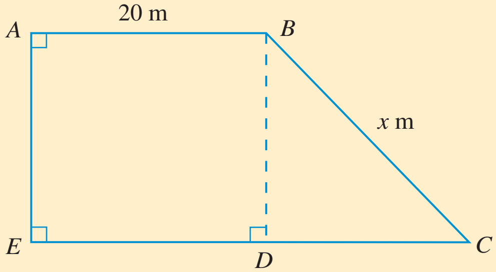
2 A square has an area of 144 cm^2. Find the length of a diagonal, correct to one decimal place.

3 A pipe is placed inside a rectangular crate, as shown. The pipe is 1.6 m long. How much of the pipe extends over the top of the crate? Give your answer in metres, correct to two decimal places.



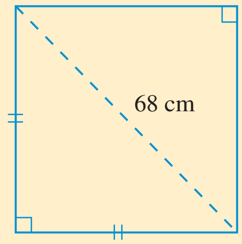
4 What is the maximum length of a steel rod that will fit inside a rectangular box whose base dimensions are 70 cm × 50 cm and whose height is 190 cm? How could the maximum length of the rod be determined?

5

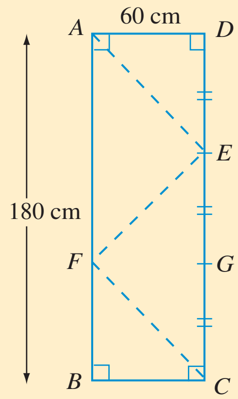


The rectangle $ABDE$ has an area of 160 m^2. Find x if ΔBDC is an isosceles triangle.

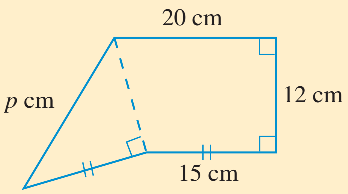
6 A square picture frame has a diagonal of 68 cm. Find the length of one side of the frame, giving your answer to the nearest centimetre.



7 Ribbon is to be sewn on a piece of material measuring 60 cm × 180 cm. The ribbon is sewn around the perimeter of *ABCD*, and then from *A* to *E* to *F* to *C*. Find (to the nearest 0.01 m) the total length of ribbon required.)



8 In the diagram on the right, find *P*, correct to one decimal place.



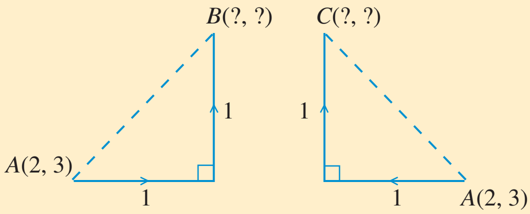
9 Here is one method of generating Pythagorean triads:

- Choose any two numbers, a and b (where $b > a$).
- Find the difference of their squares, $b^2 - a^2$.
- Find twice the product of the two numbers, $2ab$.
- Find the sum of their squares, $b^2 + a^2$.

Your results $[(b^2 - a^2), (2ab), (b^2 + a^2)]$ form a Pythagorean triad.
Generate a Pythagorean triad for each of the following pairs of numbers.

a $a = 15, b = 20$ **b** $a = 8, b = 26$ **c** $a = 4.5, b = 5.5$

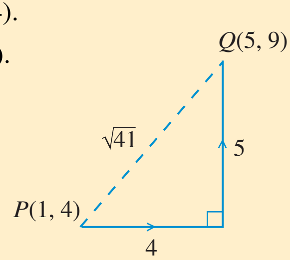
10 Using the fact that $\sqrt{2} = \sqrt{1^2 + 1^2}$ it is possible to find the end point of an interval starting at $A(2, 3)$, such that the interval is $\sqrt{2}$ units long.
What are the coordinates of *B*?



B must be $(2 + 1, 3 + 1) = (3, 4)$ and *C* must be $(2 - 1, 3 + 1) = (1, 4)$.

a Find two more points (*D* and *E*) which are $\sqrt{2}$ units from $A(2, 3)$.

b Using $\sqrt{41} = \sqrt{16 + 25} = \sqrt{4^2 + 5^2}$ then, starting at $P(1, 4)$, the point $Q(5, 9)$ is $\sqrt{41}$ units from *P*.
Find seven more points which are $\sqrt{41}$ units from $P(1, 4)$.
(*Hint*: $\sqrt{41} = \sqrt{4^2 + 5^2} = \sqrt{5^2 + 4^2}$)



Worksheet 6-08 Pythagoras' theorem crossword

Language of maths

Cartesian plane	*diagonal*	*distance*	*exact length*
hypotenuse	*interval*	*irrational number*	*number plane*
proof	*prove*	*Pythagoras' theorem*	*Pythagorean triad*
quadrilateral	*right-angled*	*side*	*square*
square root	*test*	*theorem*	*unknown*

1 What is the meaning of the word 'hypotenuse'?

2 Which word is another name for 'rule'?

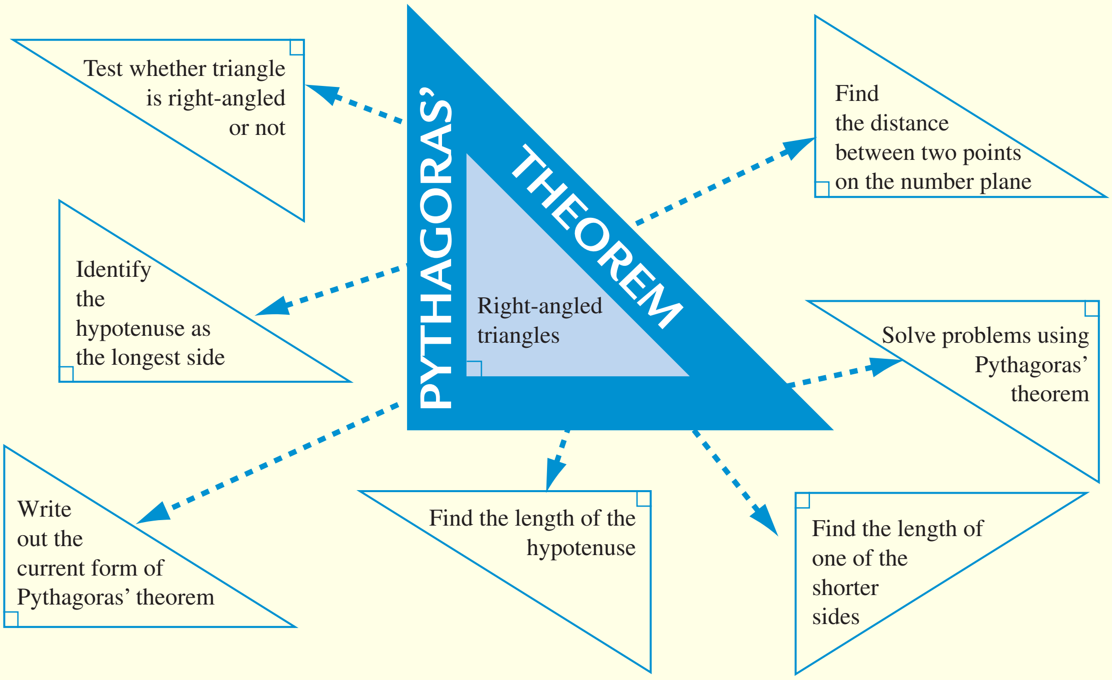
3 A diagonal is an interval from one vertex of a shape to another non-adjacent vertex. What does 'non-adjacent' vertex mean?

4 What name is given to any set of three numbers that follows Pythagoras' theorem?

5 Describe **Pythagoras' theorem** in your own words.

Topic overview

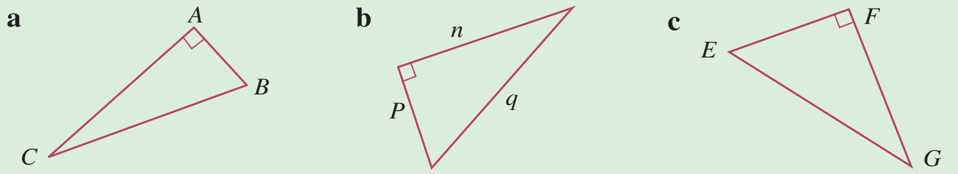
- Write, in your own words, what you have learned in this chapter.
- Write which parts of this chapter were new to you.
- Copy and complete:
  The things I understand about Pythagoras' theorem that I did not understand before are …
  The things I am still not confident in doing in this chapter are …
  (Give an example of each difficulty.)
- Copy and complete whichever applies to you:
  The steps I will take to overcome my problems with this chapter are …
  The sections of work that I found difficult in this chapter were …
  The sections of work that I found easy in this chapter were …
- Copy and complete the overview summary below.



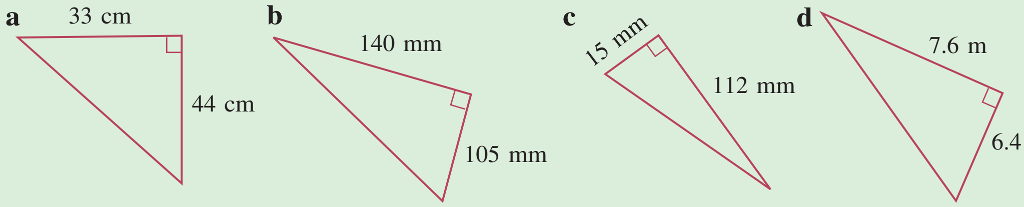
Chapter 6 Review

Topic test Chapter 6

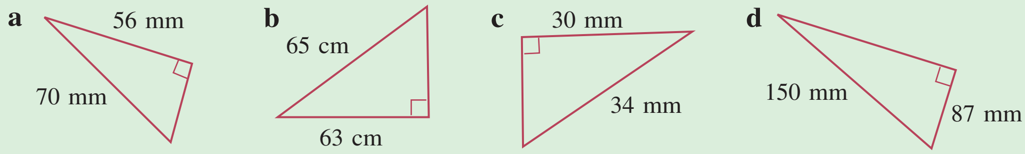
1 Name the hypotenuse in each of these triangles. Ex 6-02



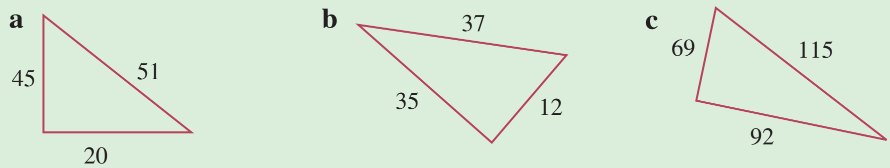
2 Calculate the length of the hypotenuse for each of these triangles. Give your answer correct to one decimal place where necessary. Ex 6-04



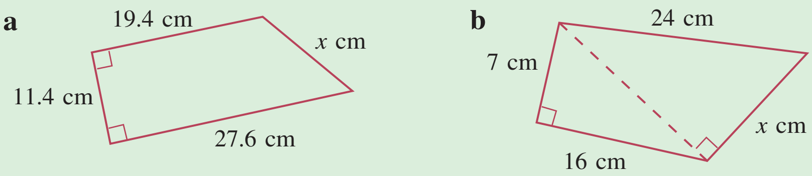
3 Calculate the length of the unknown side of each of these triangles. Give your answer correct to one decimal place where necessary. Ex 6-05



4 Decide whether each of these triangles is right-angled or not. Ex 6-06

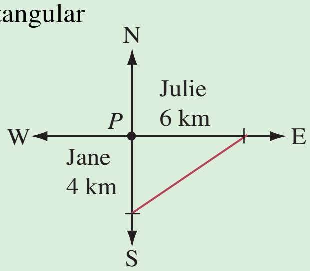


5 Find x, correct to two decimal places, in each of these quadrilaterals. Ex 6-07



6 Can an umbrella 84 cm long fit flat inside a briefcase whose rectangular base measures 48 cm × 64 cm? Ex 6-08

7 Julie walks 6 km due east from a starting point P, while Jane walks 4 km due south from P. How far are Julie and Jane apart? (Give your answer correct to one decimal place.) Ex 6-08



8 Plot the points $P(-3, -2)$ and $W(1, 5)$ on a number plane. Ex 6-09

a Draw the interval PW.

b Use the interval PW as the hypotenuse, and form a right-angled triangle.

c Write the lengths of the horizontal and vertical sides of the triangle.

d Use Pythagoras' theorem to find the length of PW (correct to one decimal place).